Fluctuations of Economic Attributes Under Monopolistic Competition

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I. Introduction

This paper examines monopolistically competitive market structures and the possible benefits of trade between two independent economies. These benefits are realized by utilizing economies of scale and increasing the number of varieties available for consumers to purchase. Monopolistic competition can be simulated in many different ways, but this research employs the model developed by Krugman (1980). Krugman developed this model to fix some of the shortcomings of "standard" trade models. Past attempts to model trade as a way to increase a nation's surplus through specialization (comparative advantage) could not explain intra-industry trade and consumer demand was not used to define international trade. Krugman's more sophisticated (and simple) model explains international trade with horizontal product differentiation, increasing returns to scale, and international trade costs.

II. Theory – Closed Economy

Consumer Problem

In monopolistic competition, each firm produces a variety of products, indexed by i. The consumer's utility function is:

$$U = \sum_i c_i^\theta \tag{1}$$

Each consumer will choose their consumption of each variety in order to maximize their own utility subject to their budget constraint:

$$\max_c \sum_i c_i^\theta \;\; s.t. \;\; Y = \sum_i p_i c_i$$

where c is consumption, $0 < \theta < 1$, p is price, and Y is income. Setting up a Lagrangian and taking the first order condition with respect to consumption will solve this simple maximization problem.

$$\mathcal{L} = \sum_i c_i^\theta + \lambda\left(Y - \sum_i p_i c_i\right)$$

$$\frac{\partial \mathcal{L}}{\partial c_i} = \theta c_i^{\theta-1} - \lambda p_i = 0$$

$$p_i = \frac{\theta c_i^{\theta-1}}{\lambda}$$

This final equation is the individual demand function. Notice that demand for variety i is independent of demand for all other varieties $j \neq i$.

In order to derive the market demand curve, we assume that each consumer represents one unit of labor. We can therefore write total consumption of each variety, C_i, as the product of individual consumption c_i and the labor stock $\bar{L}$.

$$C_i = c_i \bar{L}$$

In a closed economy, total production has to equal total consumption, q_i. Therefore

$$c_i = \frac{q_i}{\bar{L}} \tag{2}$$

By substitution this into the individual demand function we obtain the market demand curve.

$$p_i = \frac{\theta (\frac{q_i}{\bar{L}})^{\theta-1}}{\lambda}$$

Firm Problem

Labor is the only factor of production, and each firm has the same increasing returns to scale production technology, $l_i = \alpha + \beta q_i$ where l_i is the amount of labor used in industry i, and α and β are positive constants. Each firm's profit function can be written as

$$\pi_i = p_i q_i - w l_i = \frac{\theta \left(\frac{q_i}{\bar{L}}\right)^{\theta-1}}{\lambda} q_i - w(\alpha + \beta q_i)$$

where w is the wage. Each firm will maximize its profits with respect to its output. We can solve for the profit-maximizing price by taking the first order condition with respect to output:

$$\frac{\partial \pi_i}{\partial q_i} = \frac{\theta^2 q_i^{\theta-1}}{\bar{L}^{\theta-1}\lambda} - w\beta = 0$$

$$p_i = p = \frac{w\beta}{\theta} \tag{3}$$

The profit-maximizing price is independent of any variety-specific factors, so the price is the same for all varieties.

Free entry into the market implies that long-run profits equals zero. Using this condition and the profit-maximizing price in (1), we can solve for the profit-maximizing quantity of each variety:

$$p_i q_i - w l_i = 0 \tag{4}$$

$$\left(\frac{w\beta}{\theta}\right) q_i - w(\alpha + \beta q_i) = 0$$

$$q_i \left[\frac{w\beta}{\theta} - w\beta\right] = w\alpha$$

$$q_i = q = \frac{\alpha\theta}{\beta(1-\theta)}$$

Recall that individual consumption of any variety can be written as $c = q/\bar{L}$. In addition, recall that each consumer represents one unit of labor. Therefore, the labor required for production of each variety is:

$$l = c = \frac{q}{\bar{L}} = \frac{\alpha\theta}{\bar{L}\beta(1-\theta)} = \alpha + \beta q = \alpha + \frac{\alpha\theta}{(1-\theta)}$$

$$l = \alpha + \beta q = \frac{\alpha}{1-\theta} \tag{5}$$

We can finally solve for the number of varieties produced in the closed economy. If we assume full employment, then the product of the labor used for each variety, given in (3), and the number of variety should equal the labor stock:

$$vl = \bar{L} \tag{6}$$

$$v = \frac{\bar{L}(1-\theta)}{\alpha}$$

This model will be solved in GAMS using Equations 1-6.

III. Theory – Open Economy

When trade is allowed, we look at two identical economies, home (H) and foreign (F). Because the two economies are identical, relative prices will be the same in each country. Therefore, there can be no comparative advantage-based trade. However, gains from trade can occur because consumers can purchase more varieties of products. The consumer now faces a maximization problem of the form:

$$\max_c U = \sum_i c_i^\theta + \sum_j c_j^\theta \ \ s.t.\ Y = \sum_i p_i c_i + \sum_j p_j c_j$$

where i indexes the v_H varieties of home goods and j indexes the v_F varieties of foreign goods. Consumers will allocate half of their income to consumption in each country so total world demand for any variety remains unchanged. Therefore, the firm's problem remains the same and the equations for q and p hold. However, the number of varieties produced in each country is unique:

$$v_H = \frac{\bar{L}_H(1-\theta_H)}{\alpha_H}$$

$$v_F = \frac{\bar{L}_F(1-\theta_F)}{\alpha_F}$$

IV. GAMS Implementation

We implemented both the open and closed economy models in GAMS. We began by defining the parameters and setting the initial values, which can be found in Table 1.

Table 1. Initial values

α	5
β	0.5
θ	0.5
$\bar{L}$	1000
w_0	1

We then defined the variables and equations. For a closed economy, we solved the model MONOP in order to maximize utility. We then re-evaluated this model at different values of α, β, and $\bar{L}$. For an open economy, we redefined all parameters, variables, and equations to be functions of the set R, which has as its members each country in the model. In this model, the only two members of the set R are H (home) and F (foreign). We solved this open economy model TRADE maximizing a scalar objective function.

V. Results and Conclusions

We first ran the model for a closed economy given the initial parameter values found in Table 1. The results of this simulation can be found in Figure 1. The model predicts 100 firms will enter and produce in the market, each firm will produce 10 units of output, consumers will absorb .01 units of each type of output produced by different firms, the price per unit of output is 1 (which is equal to the predetermined wage rate), 10 individuals from the labor stock are used to produce the total output of each firm, and each individual in the economy has a utility index of 10. These results serve as the base

case and are compared to each of the following simulations where different parameters are altered.

Figure 1: Basic Attributes (no trade)

Variable	Description	GCE Solution
V	Varieties	100
Q	Output	10
C	Cost	0.01
P	Price	1
U	Utility Index	10
L	Labor	10
W	Wage	1

The next set of simulations (Figure 2) alters the fixed cost parameter α. For the base case, α is equal to five but is varied to 1, 2.5, 7.5, and 10 in the four other cases. We can see from the results that the number of varieties/firms in each model is inversely proportional to the level of fixed costs in the economy: the number of firms decreases as fixed costs increase. This is a direct result of economies of scale. The firms that are present in the economy when fixed costs are high posses a distinct cost advantage over the firms that are priced out of the market. Since aggregate output remains constant in each simulation, the quantity of output produced by the remaining firms increases as fixed costs increase. This maintains the fixed wage rate paid to each worker in the labor force and also explains the increase in consumption per variety for each individual in the economy. As fixed costs increase and varieties decrease, we also see an increase in the amount of labor used by each firm to preserve full employment in the economy. In each of these models, the utility index is dependent on the number of varieties and amount of each variety consumed by individuals. The magnitude of this index is proportional to the number of varieties produced in the economy and thus decreases as firms are priced out of the market.

Figure 2: Altering Fixed Cost Parameter (no trade)

Variable	α=1	α=2.5	α=5	α=7.5	α=10
V	1.75E+08	200	100	66.667	50
Q	5.71E-06	5	10	15	20
C		0.005	0.01	0.015	0.02
P	1	1	1	1	1
U	13238	14.142	10	8.165	7.071
L	5.71E-06	5	10	15	20
W	1	1	1	1	1

We also decided to examine the effects of altering the marginal cost parameter β (Figure 3). The base case included β=.5, but additional simulations were completed with β=.1, .25, .75, and .9. First notice that altering this parameter has no effect on the number of varieties, only the quantity produced by each firm. Thus, as marginal costs increase, the

quantity produced by each firm will decrease. We also see that the price per unit produced increases as marginal cost increases. This result is expected since the profit-maximizing price condition described in Equation 3 is dependent on wage, marginal cost, and the utility parameter. With the wage rate and utility parameter held constant, an increase in marginal cost implies an increase in the price level. Aggregate output does not remain constant in these simulations and decreases as marginal cost increases. This explains the decrease in individual consumption per variety since output is decreasing and the labor stock remains constant. Similar to what was found in the simulations altering fixed costs, increasing the marginal cost parameter encourages a reduction in the utility index. As noted before, the utility index is dependent on the number of varieties and amount of each variety consumed by individuals. The number of varieties in these simulations remains constant but consumption per variety declines as marginal cost increases. Thus, the utility index is negatively affected.

Figure 3: Altering Marginal Cost Parameter (no trade)

Variable	β=.1	β=.25	β=.5	β=.75	β=.9
V	100	100	100	100	100
Q	50	20	10	6.667	5.556
C	0.05	0.02	0.01	0.007	0.006
P	0.2	0.5	1	1.5	1.8
U	22.361	14.142	10	8.165	7.454
L	10	10	10	10	10
W	1	1	1	1	1

Figure 4: Altering Labor Stock (no trade)

Variable	L=300	L=650	L=1000	L=1350	L=1600
V	30	65	100	135	160
Q	10	10	10	10	10
C	0.033	0.015	0.01	0.007	0.006
P	1	1	1	1	1
U	5.477	8.062	10	11.169	12.649
L	10	10	10	10	10
W	1	1	1	1	1

The final model examining the no trade scenario alters the labor supply (Figure 4). The labor stock initially included 1000 individuals, but was changed to reflect populations of 300, 650, 1350, and 1600 workers. The number of individuals available in the market directly impacts the number of firms that are able to produce. As a result, an increase in the labor supply will increase the number of varieties present in the economy. This increase is directly proportional to the increase in the labor supply since the quantity produced by each firm remains unchanged. Since the wage rate is constant, we see the individual consumption of each variety decrease as the labor stock increases because individuals consume equal proportions of every variety available in the economy. The price of each unit of output is unchanged since the cost parameters are unaffected. The

labor used by each firm is also constant since aggregate output increases with the labor increase. We see an interesting result for these simulations as the utility index increases with an increase in the labor stock. For these cases, we see an increase in the number of varieties produced but decreased consumption per variety. Since the number of varieties available is increasing at a faster rate than consumption per variety is decreasing, an overall increase in the utility index is observed.

The next three sets of simulations introduce trade into the model (Figure 5), alters the utility parameter (Figure 6), and looks at the effects of disproportionate labor forces between the two economies (Figure 7). The results in Figure 5 details the results of two economies trading finished goods under the initial parameters described in Table 1. In the figure's final column "Trade," the given values represent each variable's equilibrium level in the home (H) and foreign (F) economies. We see that the number of varieties, output per firm, price per unit, labor used by each firm, and wage rate remain the same when trade is promoted between each economy. Consumption per variety is exactly half the level of what was found in the closed economy model because individuals will now spend half their income on varieties produced in the home county and half on those produced in the foreign market. We also see an increase in the utility index when trade is introduced. For this case, we again see the number of varieties produced has doubled and the fixed wage level forces consumption per variety to decrease. Since the number of varieties available is increasing at a faster rate than consumption per variety is decreasing, an overall increase in the utility index is observed.

Figure 5: Basic Attributes (no trade vs. trade)

Variable	Description	No Trade	Trade
V	Varieties	100	100
Q	Output	10	10
C	Cost	0.01	0.005
P	Price	1	1
U	Utility Index	10	14.142
L	Labor	10	10
W	Wage	1	1

The utility parameter can be interpreted as an individual's preference to consume as much as possible of a single variety. In other words, individuals with a low utility parameter do not have preferences for a specific variety and are willing to consume available substitutes. As the utility parameter increases, individuals prefer fewer varieties and the availability of substitutes is less desirable. We can see this interpretation in the simulations described by Figure 6. In the first model where trade is introduced, the utility parameter of the home country is θ_H=.5 (equal to θ_F). The additional simulations in Figure 6 summarize the effects of changing θ_H (.1, .25, .75, and .9) while θ_F remains constant. As predicted, the utility parameter is proportional to the number of varieties produced in the home county: the number of producing firms increases when θ_H is small. As θ_H increases, the number of firms decreases and consumption per variety increases.

The drop in the domestic price level is also expected since the utility parameter is inversely proportional to the price (Equation 3).

Figure 6: Altering Utility Parameter (trade)

	θ_H=.1		θ_H=.25		**θ_H=.5**		θ_H=.75		**θ_H=.9**	
Variable	**H**	**F**	H	F	**H**	**F**	H	F	**H**	**F**
V	**180**	**100**	150	100	**100**	**100**	50	100	**20**	**100**
Q	**1.111**	**10**	3.33	10	**10**	**10**	30	10	**90**	**10**
C	**6E-05**	**0.005**	0.002	0.005	**0.005**	**0.005**	0.015	0.005	**0.045**	**0.005**
P	**5**	**1**	2	1	**1**	**1**	0.667	1	**0.556**	**1**
U	**132.32**	**19.799**	50.131	17.678	**14.142**	**14.142**	6.429	10.607	**7.363**	**8.485**
L	**5.556**	**10**	6.667	10	**10**	**10**	20	10	**50**	**10**
W	**1**	**1**	1	1	**1**	**1**	1	1	**1**	**1**

Figure 7: Altering Foreign Labor Supply (trade)

	1000/1000		1000/750		**1000/500**		1000/250		**1000/100**	
Variable	**H**	**F**	H	F	**H**	**F**	H	F	**H**	**F**
V	**100**	**100**	100	75	**100**	**50**	100	25	**100**	**10**
Q	**10**	**10**	10	10	**10**	**10**	10	10	**10**	**10**
C	**0.005**	**0.005**	0.006	0.006	**0.007**	**0.007**	0.008	0.008	**0.009**	**0.009**
P	**1**	**1**	1	1	**1**	**1**	1	1	**1**	**1**
U	**14.142**	**14.142**	13.229	13.229	**12.247**	**12.247**	11.18	11.18	**10.488**	**10.488**
L	**10**	**10**	10	10	**10**	**10**	10	10	**10**	**10**
W	**1**	**1**	1	1	**1**	**1**	1	1	**1**	**1**

The final model examining the trade scenario alters the labor supply in the foreign country while the domestic stock is unchanged (Figure 7). The price for one unit of each variety, the quantity of labor used by each firm, and the wage rate in both countries are unaffected by a reduction in the foreign labor supply. As seen in the no trade scenarios, the number of individuals available in the market directly impacts the number of firms that are able to produce and a decrease in the labor supply will decrease the number of varieties present in the economy. Consumption per variety increases as the aggregate number of varieties decreases since we assume individuals spend their entire wage. The fluctuation in the utility index (Figure 7) summarizes the most important result from all of the previous simulations: opening an economy to trade increases the utility of individuals in both countries. For this last model, the foreign and domestic utility indices decrease since the labor supply in the foreign economy decreases at a faster rate then the consumption per variety is increasing. The foreign labor supply has decreased by 90% in the last column of Figure 7, but the utility index for each country is still higher than our base case levels found in Figure 1. We can see similar results when comparing these values to the outcomes in Figure 4: an economy will always see gains from trade when conducting trade with a larger economy. The larger economy participating in trade will also benefit, but the effects are not as pronounced.

Appendix

GAMS Code – Closed Economy

```
$OFFLISTING
OPTION LIMROW=0;
OPTION LIMCOL=0;

PARAMETERS
THETA
LBAR
ALPHA
BETA
WO
VO
QO
CO
PO
UO
LO;

ALPHA=5;
BETA=.5;
THETA=.5;
LBAR=1000;
WO=1;
VO=LBAR*(1-THETA)/ALPHA;
QO=ALPHA*THETA/(BETA*(1-THETA));
CO=QO/LBAR;
LO=ALPHA+BETA*QO;
PO=BETA*WO/THETA;
UO=VO*CO**THETA;

VARIABLES
V
Q
C
P
U
L
W;

W.FX=WO;
V.LO=0; Q.LO=0; C.LO=0; P.LO=0; L.LO=0;
V.L=VO; Q.L=QO; C.L=CO; P.L=PO; L.L=LO; U.L=UO;

EQUATIONS
UTILITY
CLEARING
PRICE
COST
ENTRY
RESOURCE;
```

```
UTILITY..U=E=V*C**THETA;
CLEARING..C=E=Q/LBAR;
PRICE..P=E=BETA*W/THETA;
COST..L=E=ALPHA+BETA*Q;
ENTRY..P*Q-L*W=E=0;
RESOURCE..LBAR=E=V*L;

MODEL MONOP /ALL/;
SOLVE MONOP USING NLP MAXIMIZING U;

ALPHA=0;
SOLVE MONOP USING NLP MAXIMIZING U;
ALPHA=2.5;
SOLVE MONOP USING NLP MAXIMIZING U;
ALPHA=7.5;
SOLVE MONOP USING NLP MAXIMIZING U;
ALPHA=10;
SOLVE MONOP USING NLP MAXIMIZING U;

BETA=0.1;
SOLVE MONOP USING NLP MAXIMIZING U;
BETA=.25;
SOLVE MONOP USING NLP MAXIMIZING U;
BETA=.75;
SOLVE MONOP USING NLP MAXIMIZING U;
BETA=.9;
SOLVE MONOP USING NLP MAXIMIZING U;

LBAR=300;
SOLVE MONOP USING NLP MAXIMIZING U;
LBAR=650;
SOLVE MONOP USING NLP MAXIMIZING U;
LBAR=1350;
SOLVE MONOP USING NLP MAXIMIZING U;
LBAR=1600;
SOLVE MONOP USING NLP MAXIMIZING U;
```

GAMS Code – Open Economy

```
$OFFLISTING
OPTION LIMROW=0;
OPTION LIMCOL=0;

SET R /H,F/;
ALIAS(R, RR);

PARAMETERS
THETA(R)
LBAR(R)
ALPHA(R)
BETA(R)
WO(R)
```

```
VO(R)
QO(R)
CO(R)
PO(R)
UO(R)
LO(R);

ALPHA(R)=5;
BETA(R)=.5;
THETA(R)=.5;
LBAR(R)=1000;
WO(R)=1;
VO(R)=LBAR(R)*(1-THETA(R))/ALPHA(R);
QO(R)=ALPHA(R)*THETA(R)/(BETA(R)*(1-THETA(R)));
CO(R)=QO(R)/LBAR(R);
LO(R)=ALPHA(R)+BETA(R)*QO(R);
PO(R)=BETA(R)*WO(R)/THETA(R);
UO(R)=VO(R)*CO(R)**THETA(R);

VARIABLES
V(R)
Q(R)
C(R)
P(R)
U(R)
L(R)
W(R)
VT(R)
OBJ;

W.FX(R)=WO(R);
V.LO(R)=0; Q.LO(R)=0; C.LO(R)=0; P.LO(R)=0; L.LO(R)=0;
V.L(R)=VO(R); Q.L(R)=QO(R); C.L(R)=CO(R); P.L(R)=PO(R); L.L(R)=LO(R); U.L(R)=UO(R);

EQUATIONS
UTILITY(R)
CLEARING_A(R)
CLEARING_T(R)
VARIETY_A(R)
VARIETY_T(R)
PRICE(R)
COST(R)
ENTRY(R)
RESOURCE(R)
OBJECTIVE;

UTILITY(R)..U(R)=E=VT(R)*C(R)**THETA(R);
CLEARING_A(R)..C(R)=E=Q(R)/LBAR(R);
CLEARING_T(R)..C(R)=E=Q(R)/SUM(RR, LBAR(RR));
VARIETY_A(R)..VT(R)=E=V(R);
VARIETY_T(R)..VT(R)=E=SUM(RR, V(RR));
PRICE(R)..P(R)=E=BETA(R)*W(R)/THETA(R);
COST(R)..L(R)=E=ALPHA(R)+BETA(R)*Q(R);
ENTRY(R)..P(R)*Q(R)-L(R)*W(R)=E=0;
```

```
RESOURCE(R)..LBAR(R)=E=V(R)*L(R);
OBJECTIVE..OBJ=E=0;

MODEL AUTARKY /UTILITY  CLEARING_A VARIETY_A PRICE COST ENTRY RESOURCE
OBJECTIVE/;
SOLVE AUTARKY USING NLP MAXIMIZING OBJ;


MODEL TRADE /UTILITY CLEARING_T VARIETY_T PRICE COST ENTRY RESOURCE
OBJECTIVE/;

SOLVE TRADE USING NLP MAXIMIZING OBJ;



LBAR('H')=1000; LBAR('F')=700;
SOLVE TRADE USING NLP MAXIMIZING OBJ;


MODEL TRADE /UTILITY CLEARING_T VARIETY_T PRICE COST ENTRY RESOURCE
OBJECTIVE/;
THETA('H')=.1;
SOLVE TRADE USING NLP MAXIMIZING OBJ;
THETA('H')=.25;
SOLVE TRADE USING NLP MAXIMIZING OBJ;
THETA('H')=.75;
SOLVE TRADE USING NLP MAXIMIZING OBJ;
THETA('H')=.9;
SOLVE TRADE USING NLP MAXIMIZING OBJ;
```