Midterm Project:
Monetary Policy and Propensity to Hoarding

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Content

1 Introduction

In Tsiang(1961) [5]'s model, he proved the crucial factor in analyzing devaluation effect on trade balance lies in the propensity to hoarding via changes in income and interest rate. That is individuals' preference on storing part of their income might make the additional consumption small when national income increases. Therefore, under Marshall-Lerner condition, a positive trade balance after devaluation would imply preference to hoarding and a negative trade balance would imply dishoarding. However, in that paper, Tsiang (1961) [5] failed to explain the reason for an individual to hoard money. Money demand function is simply a linear function of income and real interest rate with constant propensity to hoarding. Therefore, money is rather a byproduct of an increasing income in a transaction-cost free society.

However, as Brunner and Meltzer (1967) [2] pointed out the propensity to hoarding should not be constant, yet changing with the volume of transactions. Under such assumption, money is not only a measure of wealth, but also a media for transaction. Saving (1971) [3] used Brunner and Meltzer's idea on transaction cost, deriving from individual's utility maximization behavior without incorporating money holdings directly in utility function. In Saving's framework, he divided an agent time to leisure, working and transaction. Such transaction time is a nonlinear function of money holding and consumption goods. Therefore, by incorporating time constraint and a budget constraint in individual's utility maximizing behavior, Saving got a money demand function with changing propensity to hoarding and make money holding a rational behavior for an agent.

So, in this project, we first will explore Saving's model and find how money demand would be altered by consumption, production technology, labor-leisure choice and capital accumulation. We then will find some fiscal and monetary policy implications on that model. Secondly, in our final project, we will extend this model into two-country two-goods economy, to see how devaluation would affect trade balance via changing in propensity of hoarding. In section II, we briefly introduced and model; section III we compute the steady state of the model; section IV we did simulation and discuss the changing of fiscal and monetary policy. Section V we provide some future extension.

2 The Model Closed Economy

2.1 Household

The economy has large amount of identical individual. The representative agent maximize his life time utility by choosing amount of consumptions $c_t \geq 0$ and leisure $l_t \geq 0$:

$$\sum_{t=0}^{\infty} \beta^t U(c_t, l_t) = \sum_{t=0}^{\infty} \beta^t [\frac{c_t^{1-\rho}}{1-\rho} + \frac{l_t^{1-\eta}}{1-\eta}] \tag{1}$$

Here $\beta \in (0,1)$ is the discounted rate for future consumption and leisure, it can also be viewed as subjective interest rate required by an agent. ρ is the inverse of elasticity of intertemporal substitution (EIS) of consumption goods and η is the inverse of EIS of leisure. In general, EIS measures the percentage of changes in consumption (leisure) in response to interest rate. A lower value of ρ (η) means consumption (leisure) growth is more sensitive to changes in interest rate (higher elasticity). That is, when interest rate increase, the agent with higher EIS would put large amount of money in saving. So higher elasticity indicate a lower incentive for consumption (leisure) smoothing.

Such setting of utility function also satisfies a diminishing marginal utility of consumption and leisure: $U_c > 0$,$U_l > 0$,$U_cc < 0$ and $U_ll < 0$. The separation of leisure and consumption in utility also let income effect and substitution effect from wealth changes cancelled out. Therefore, a constant workforce in balanced line is guaranteed.

Further, we assume in each period, this agent will divide his time into three portion: working (n_t), leisure (l_t) and shopping (s_t). The shopping time is a function of consumption and real money balance (m_{t+1}/p_t). Therefore, we have a time constraint for the representative agent:

$$l_t + n_t + s_t = l_t + n_t + \frac{c_t}{1 + m_{t+1}/p_t} \leq 1 \tag{2}$$

The above shows $s_t = H(c_t, m_{t+1}/p_t) = \frac{c_t}{1+m_{t+1}/p_t}$, satisfying: $H_{c_t} > 0$ and $H_{m_{t+1}/p_t} < 0$. Higher consumption requires more shopping time and more money holding reduces shopping time. That is, the more holding of money can reduce our bargaining times, therefore, reduce the time we spend on shopping.

Besides, we assume the agent owns the firm and each period, he provide labor and capital to the firm at wage w_t and rental rate r_t, respectively, and receive firm's profit of Π_t in the end of period t. The agent maximize his utility also subject to a budget constraint:

$$c_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} + k_{t+1} - (1-\delta)k_t \leq w_t n_t + r_t k_t + b_t - \tau_t + \frac{m_t}{p_t} + \Pi_t \tag{3}$$

where the left hand side is the expenditure for that agent in period t and the right hand side is agent's wealth at the beginning period t. Each period, the agent's wealth contains six parts: wage w_t received by providing labor; income r_t received by providing capital; firm's profit Π_t; matured bond b_t bought at period $t-1$; the nominal balance held between $t-1$ and t, m_t is chosen in period $t-1$, evaluated by p_t in time t; and a lump-sum tax τ_t paid to government. The expenditure includes: consumption c_t; nominal balance m_{t+1}, held between t and $t+1$ for purchasing c_t, evaluated by price p_t; one-period government bond with real gross interest rate of return R_t and capital investment $k_{t+1} - (1-\delta)k_t = I_t$ for accumulating capital in the next period. So the agent's problem is to maximize (1), subject to (2) and (3) by choosing c_t, l_t, n_t, b_{t+1}, m_{t+1} and k_{t+1}. We have the F.O.C as:

$$[l_t]: \quad U_{l_t} = \mu_t \tag{4}$$

The shadow value of time equals to the marginal utility of leisure. This is due to our assumption of an utility function with separate consumption and leisure.

$$[c_t]: \quad \lambda_t = U_{c_t} - \mu_t s_{c_t} = U_{c_t} - U_{l_t} H_{c_t} \tag{5}$$

The shadow price of wealth equals to the marginal utility of consumption less the marginal disutility(sacrifice for leisure time) for shopping that consumption. That is once the agent receive extra income, such income would not improve his utility proportionally, but also cause him some time to shopping, which reduce his utility from leisure.

$$[n_t]: \quad \lambda_t w_t = \mu_t = U_{l_t} \tag{6}$$

Agent choose labor input to make the utility of leisure equals to the shadow value of wealth times the wage rate. This indicates either increase in shadow value or wage rate will increase leisure through the welfare effect.

$$[k_{t+1}]: \quad \beta\lambda_{t+1}[r_{t+1}+1-\delta] = \lambda_t \tag{7}$$

$$[b_{t+1}]: \quad \beta\lambda_{t+1}R_t = \lambda_t \tag{8}$$

Combining these two equations, we can have: $R_t = r_{t+1} + 1 - \delta$. That is the gross return in bond should equal to gross return on capital less the depreciation rate. Also in equilibrium, when $\lambda_t = \lambda_{t+1}$, $R_t = 1/\beta$. That is to say the objective gross return should equal to the subjective gross return required by agent.

$$[m_{t+1}]: \quad \beta\lambda_{t+1}\frac{1}{p_{t+1}} = \lambda_t\frac{1}{p_t} + \mu_t\frac{1}{p_t}H_{m_{t+1}/p_t} \tag{9}$$

Let $Rm_t = p_t/p_{t+1}$, an inverse of inflation rate, reflecting the real gross return on holding money. Rearrange equation (9) by substitute out λ_t, λ_{t+1} and μ_t we can have:

$$\frac{R_t - Rm_t}{R_t}[U_{c_t} - U_{l_t}H_{c_t}] = -U_{l_t}H_{m_{t+1}/p_t} \tag{10}$$

The above equations shows the tradeoff between holding money and holding bonds. The increase in money holding will decrease the shopping time and therefore improve agent's utility via an increase in leisure. However, holding more money instead of bonds will lose the real gross return on bonds and therefore, lose the utility gained from welfare effects on increasing consumption $U_{c_t} - U_{l_t}H_{c_t}$. So, the agent's choice of money holding should equalize benefit and cost of money holding.

2.2 Firm

For firm sectors, we also have firm's profit maximizing behavior under competitive market. Therefore, we know the total profit of a competitive firm should be $\Pi_t = 0$. Further we assume the firm follows a constant return to scale Cobb-Douglas production function with labor output elasticity of α: $Y_t = A_t N_t^{\alpha} K_t^{1-\alpha}$. And the firm's maximizing behavior determines the wage rate and capital rental rate as:

$$\begin{aligned} w_t &= \alpha A_t (N_t/K_t)^{\alpha-1} \\ r_t &= (1-\alpha)A_t(N_t/K_t)^{\alpha} \end{aligned} \tag{11}$$

2.3 Government

Government will finance its expenditure g_t by imposing a lump-sum tax on household, issuing bonds and printing money. So in each period, government will subject to a budget constraint:

$$g_t = \tau_t + \frac{B_{t+1}}{R_t} - B_t + \frac{M_{t+1-M_t}}{p_t} \tag{12}$$

B_t is the bond the government issued to the agent at period $t-1$, maturing at the beginning of period t. M_{t+1} is the stock of currency government issued at the end of period t and M_t is the stock of currency government retired at the beginning of period t. Given the initial level of B_0 and M_0, in each period, the government can choose g_t, t_t and B_t.

3 Steady State Analysis

We define a competitive equilibrium as taken a exogenous sequence $\{g_t, \tau_t\}$, a sequence of price $\{p_t, w_t, r_t, R_t\}_{t=0}^{\infty}$, a sequence of allocation $\{c_t, k_t, n_t, l_t, b_t, m_t\}_{t=0}^{\infty}$, such that 1) the sequence $\{c_t, I_t, n_t, l_t, b_t, m_t\}_{t=0}^{\infty}$ solves the household problem given price $\{p_t, w_t, r_t, R_t\}_{t=0}^{\infty}$, 2) the sequence $\{N_t, K_t\}$ solves the firm's problem given price $\{p_t, w_t, r_t\}_{t=0}^{\infty}$ and 3) the sequence $\{B_t, M_t, g_t, \tau_t\}_{t=0}^{\infty}$ satisfies government budget constraint for all $t \geq 0$. Market clear with $n_t = N_t$, $k_t = K_t$, $B_t = b_t$ and $M_t = m_t$ for each period. The resource constraint satisfied: $Y_t = g_t + c_t + I_t$ for each period.

We define the equilibrium for government budget constraint in the short run (initial date, with $t = 0$) and long run (when $t \geq 1$). In the long run we have $\tau_t = \tau$, $B_t = b_t = b$, $M_{t+1}/p_t = m_{t+1}/p_t = m/p$, $g_t = g$ and in the short run we have $b_t = b_0 \neq b$, $m_t = m_0 \neq m$, and $\tau_t = \tau_o \neq \tau$. Therefore, the system we derived subject to different government budget constraint in the short run and long run. We then have our stationary equilibrium system, define the value of c, n, k, l, m/p, Rm, p_0, R by eight equations:

$$
\begin{aligned}
& g - \tau - (\frac{1}{R} - 1)b = \frac{m}{p}(1 - Rm) \\
& g - \tau_0 + b_0 = \frac{m}{p} - \frac{m_0}{p_0} - \frac{b}{R} \\
& n + l + (1 + \frac{m}{p})^{-1}c = 1 \\
& c + \delta k + g = An^{\alpha}k^{1-\alpha} \\
& (1-\alpha)A(\frac{n}{k})^{\alpha} = R - 1 + \delta \\
& c^{-\rho}l^{\eta} - (1 + \frac{m}{p})^{-1} = \frac{1}{\alpha A}(\frac{n}{k})^{1-\alpha} \\
& (c^{-\rho}l^{\eta} - (1 + \frac{m}{p})(\frac{Rm}{R} - 1) = -(1 + \frac{m}{p})^{-2} \\
& R = \beta^{-1}
\end{aligned}
\tag{13}
$$

Rearranging the above equations, and we can have a constant labor-capital ratio defined as: $\gamma = \frac{n}{k} = [\frac{R-1+\delta}{(1-\alpha)A}]^{\frac{1}{\alpha}}$. The the money demand equation can be expressed by a reduced form:

$$
\frac{m}{p} = f(Rm) = (\frac{\gamma^{1-\alpha}}{\alpha A})^{-\frac{1}{2}}(1 - \frac{Rm}{R})^{-\frac{1}{2}} - 1 \tag{14}
$$

Therefore, the real money balance is an increasing function of Rm/R, the inverse of nominal interest rate [1], which is consistent with most monetary theory. In equilibrium, the real interest rate is constant, therefore, the real money balance is a decreasing function of inflation. When the inflation rate is higher, the agent would be more reluctant to hold liquidity, sacrificing interest rate he would received by holding bonds(Baumol and Tobin, 1989) [1].

Therefore, with long term government budget constraint: $g - \tau - (\frac{1}{R} - 1)b = f(Rm)(1 - Rm)$, with exogenous policy parameter g, τ, b and a stationary real gross interest rate R, the real return on money Rm can be pinned down. Here we define $g - \tau - (\frac{1}{R} - 1)b$ as gross of interest government deficit. And $g - \tau$ as net of interest deficit, also can be

[1] by Fisher Equation, we know nominal interest rate equals to real interest rate times inflation rate.

regarded as the operational deficit. That is, if the government want to peg the inflation rate, it can use fiscal policy to achieve a certain level of inflation rate. $f(Rm)(1 - Rm)$ is the seigniorage revenue from printing money, which is a product of inflation tax rate $1 - Rm$ and the quantity of real balance $f(Rm)$, the base of the inflation tax. Therefore, the government can finance its fiscal expenditure by lump sum tax, bond issuing (net of interest paid) and seigniorage revenue generated by printing money. The determination of Rm is illustrated in figure 1. Once the long run inflation rate is determined, the current price level can also be pinned down, given initial bond b_0 and initial money stock m_0, by the current government budget constraint: $m_0/p_0 = f(Rm) - (g - t_0 + b_0) + b/R$.

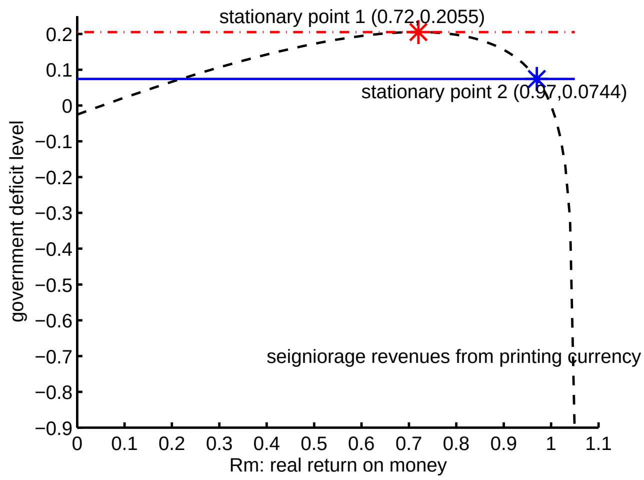

Figure 1: Determination of Rm

By solving the above system, setting government expenditure as: $g = 0.2$, $\tau = 0.15$, $\beta = 0.95$ and $A = 1.3$ and $b = 0.488$; the initial bond and money stock as: $b_0 = 3$ and $m_0 = 2$. Here we discuss the monetary policy as the open market sale to peg inflation rate in stationary equilibrium. We analyzed two situation: 1) the government maximize its seigniorage revenue via setting inflation rate; and 2) the government peg a low inflation rate. We then have:

Comparing the two columns, we can find the as government pursuing maximized seigniorage revenue, it will increase inflation rate via issuing bonds and retiring real money supply. The open market sale of bonds increased the inflation rate from 3% into 39%, with a huge decline in real money balance (from 2.48 to 0.74) and a decline in consumption (from 0.49 to 0.26). That is, in a hyperinflation society, as the inflation rate increased by bond issuing, the agent will choose to have more bonds and reduce its consumption, and therefore, leisure increased, the amount of time devoted to work reduced, capital accumulation and output reduced consequently. Since the capital-labor ratio is constant due to the setting of our production technology, the wage rate and the capital rental rate would be constant. Therefore, the decrease in money supply accompanied by an open market operation of bond issuing will increase the initial price level a lot and also increase the inflation rate permanently, which brings out a large harm on the economy.

Table 1: Equilibrium Value

Variable	Situation 1)	Situation 2)
consumption c	0.4914	0.2579
labor input n	0.6843	0.4532
capital input k	0.7902	0.5233
leisure l	0.1746	0.3981
gross real interest rate R	1.0526	1.0526
real return on capital r	0.3526	0.3526
wage w	0.9501	0.9501
real money balance m/P	2.4800	0.7339
real return on money Rm	0.9700	0.7200
inflation rate π	0.0309	0.3889
bond issued b	0.4880	3.1100
seigniorage revenue $f(Rm)(1-Rm)$	0.0744	0.2055
initial price level p_0	0.0231	11.6764

4 Simulation and Analysis

In this section, we analysis percentage change in exogenous policy variable around stationary equilibrium (1) we get in section III. We differentiated the system (13) around steady state, can get:

$$\begin{aligned}
g\hat{g} - \tau\hat{\tau} + \frac{b}{R}\hat{R} - (\frac{1}{R} - 1)b\hat{b} &= \frac{m}{p}(1 - Rm)(\frac{\hat{m}}{p}) - \frac{m}{p}Rm\hat{Rm} \\
g\hat{g} - \tau_0\hat{\tau}_0 + b_0\hat{b}_0 &= \frac{m}{p}(\frac{\hat{m}}{p}) - \frac{m_0}{p_0}(\hat{m}_0 - \hat{p}_0) - \frac{b}{R}(\hat{b} - \hat{R}) \\
n\hat{n} + l\hat{l} + (1 + \frac{m}{p})^{-1}c\hat{c} - (1 + \frac{m}{p})^{-2}c\frac{m}{p}(\frac{\hat{m}}{p}) &= 0 \\
c\hat{c} + \delta k\hat{k} + g\hat{g} &= An^{\alpha}k^{1-\alpha}[\alpha\hat{n} + (1 - \alpha)\hat{k} + \hat{A}] \\
\alpha\hat{n} + \hat{A} - \alpha\hat{k} &= R\hat{R} \\
\hat{w} &= \alpha A + (\alpha - 1)\hat{n} + (1 - \alpha)\hat{k} \\
\hat{r} &= \alpha\hat{A} + \alpha\hat{n} - \alpha\hat{k} \\
c^{-\rho}l^{\eta}[-\rho\hat{c} + \eta\hat{l}] + \frac{m}{p}(1 + \frac{m}{p})^{-2}(\frac{\hat{m}}{p}) &= \frac{\eta}{\alpha A}(\frac{n}{k})^{1-\alpha}[-\hat{A} + (1 - \alpha)\hat{n} - (1 - \alpha)\hat{k}] \\
-\hat{A} + (1 - \alpha)\hat{n} - (1 - \alpha)\hat{k} + \frac{Rm}{Rm - R}(\hat{Rm} - \hat{R}) &= -2(1 + \frac{m}{p})^{-1}\frac{m}{p}(\frac{\hat{m}}{p})
\end{aligned} \tag{15}$$

In section III, we change the policy variable b to target inflation rate, either pursuing lower inflation rate or maximizing seigniorage revenue. However, in this section, we treat all policy variable exogenous (g,b and τ), examining changes in endogenous variable caused by changes in policy variable. In this case, those changes in endogenous variable will not be anticipated by agents and the economy will be ended in disequilibrium. Table 2 display those effects.

4.1 Policy 1: changes in government expenditure

According to table 2, 1% increase in government expending will have the following results:

Table 2: Transition Matrix Around Low Inflation Rate ($Rm = 0.97$)

Variable	$\hat{A}$	$\hat{g}$	$\hat{t}$	$\hat{t}_0$	$\hat{b}$	$\hat{b}_0$	$\hat{m}_0$	R
c	1.9269	-0.6179	-0.0249	0.0000	0.0041	0.0000	0.0000	0.1235
n	-0.3508	-0.0897	-0.0160	0.0000	0.0026	0.0000	0.0000	0.2491
k	3.3555	-0.6179	-0.0249	0.0000	0.0041	0.0000	0.0000	-1.3802
l	1.3943	-0.2264	0.8914	0.0000	-0.1450	0.0000	0.0000	-2.4992
r	-1.5944	0.3698	0.0062	0.0000	-0.0010	0.0000	0.0000	1.1405
w	2.1119	-0.1585	-0.0027	0.0000	0.0004	0.0000	0.0000	-0.4888
m/P	1.3449	-0.9188	0.6891	0.0000	-0.1121	0.0000	0.0000	-1.2127
Rm	0.0416	-0.1116	0.0837	0.0000	-0.0136	0.0000	0.0000	-0.2302
p_0	-0.0386	0.0287	-0.0198	-0.0016	0.0086	0.0347	1.0000	0.0294

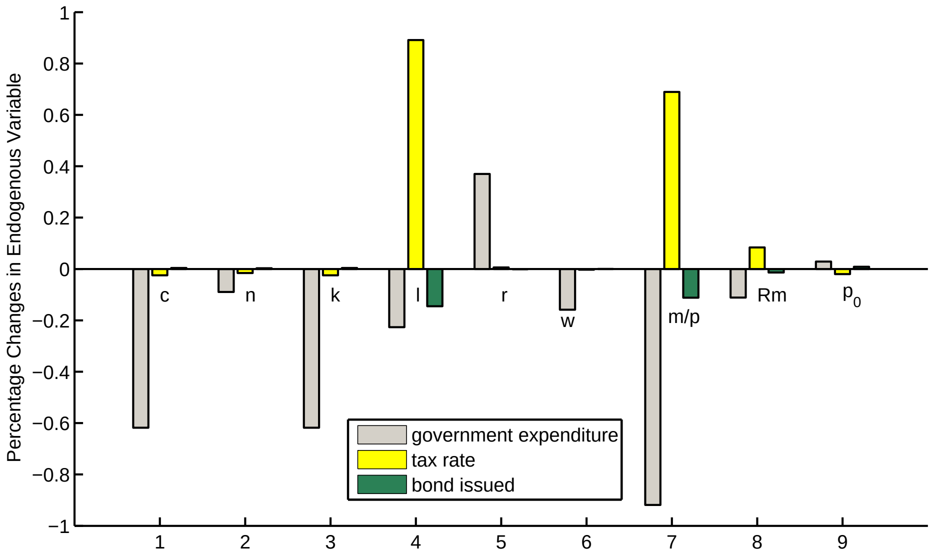

Figure 2: Percentage Changes in Government Expenditure, Tax Rate and Bond Issued

- Decrease consumption c and capital input k by 0.6%. Based on resource constraint: $Y = C + I + G$, the increase in government expenditure will crow out private expenditure c by the same proportion if we hold national income Y constant. Since c decrease smaller than the increase in g and also I decrease, the total output will also decrease. As the capital output elasticity is small, capital would be largely decreased.

- Real money demand decrease significantly by 0.9%. The increase in government expenditure will be financed by seigniorage revenue via increasing in inflation rate, therefore the real return on holding money will decrease. Therefore, decrease the real money demand.

- Decrease in leisure and labor input. The increase in government expenditure crow out consumption and then decrease the output. Therefore, labor and capital should also decrease. On the other hand, since the decrease in real money demand is larger than the decrease in consumption, so the total shopping time will increase, which results in a decrease in leisure time.

- Increase real interest rate r by 0.35%, decrease wage w by 0.16%. Since the increase in government expenditure will decrease the labor-capital ratio, wage will be relatively cheaper than capital. Therefore, the real interest rate r increase and real wage decrease.

4.2 Policy 2: changes in lump sum tax

Given a 1% increase in lump sum tax t, the outcomes are as follows:

- Increase the real money demand m/p by 0.7%. And the real return on money holding increase, and the inflation will decrease. The increase in lump sum tax will decrease governments' incentive to finance its expenditure by seigniorage revenue, therefore, government need small seignorage and increase the real return on money holding. Thus, it encourage agent hold more money.

- A 0.9% increase in leisure $\hat{l}$. Since the agent holds more money and the consumption and output merely changed. Agent will spend less time shopping around. Therefore, the leisure time will increase a lot.

4.3 Policy 3: changes in bond issuing

1% increase in bond issued $\hat{b}$ will produce following results:

- Decrease in real money factor (m/p) and a small decrease in real return on money Rm. When in the long run the government increase its debt level, and remain fiscal policy constant. It will need higher seignorage and pay back its debt. Therefore, the inflation rate should increase and the real return on money holding decrease. As the real return decrease, the agent will have less incentive to hold money, shrinking real money stocks.

- Decrease in leisure by 0.15%. Since the agent hold less money, and the consumption and output almost remain unchanged, the agent will spend more time on shopping. Therefore, leisure time will decrease.

The 1% change in lump sum tax or government expenditure has a more drastic effect on the changes in leisure and real money holding, compared to that of the change in bonds. Therefore, fiscal policy would have more direct influence than that of monetary policy.

4.4 External Technology Shocks

A 1% change in positive technology shocks on endogenous variable will be large, producing following results:

- Increase consumption and real money holding. The increase in technology will increase output therefore increase consumption. The increase in consumption will in turn increase agents' demand on real money holding. Since capital is more productive and have less output elasticity, capital will increase more than labor.

- Decrease labor input, increase capital stock and increase leisure. The increase in technology will decrease the labor-capital ratio if holding real interest rate constant. Therefore, labor will decrease and capital input will decrease a lot, as labor is more productive than capital. Such decrease in labor will increase leisure time.

- Decrease capital rental rate and increase wage rate. As labor-capital ratio decrease, capital rental return will be dragged down, pushing wage rate up.

4.5 External Interest Rate Change

A 1% increase in real gross interest rate will have great impact on capital, leisure, capital return and money holding.

- Decrease real money holding and real return on money. Increase in interest rate will increase the return on bonds and therefore the household will decrease monetary holding as the consumption increase a little. The decrease in demand of real money stock will then drag down the real return on money.
- Decrease capital accumulation and leisure. The increase in interest rate will increase labor-capital ratio once holding technology unchanged. Therefore, labor input will increase and capital stock will decrease. As labor increase and money holding decrease, agent will spend more time shopping and working, decreasing his leisure time a lot.
- Increase capital rental rate and decrease wage rate. The increase in labor-capital ratio will increase the capital rental rate and decrease wage rate as function (11) indicates.

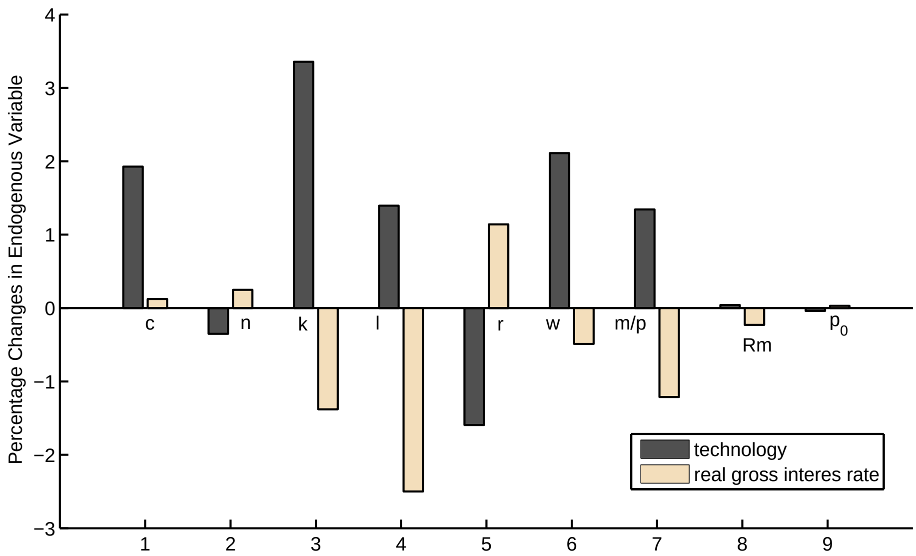

Figure 3: Percentage Changes in Technology and Real Gross Interest Rate

5 Extension

- Growth of Economy:
  In our paper, we assume a constant economic growth rate, however, we can modify our paper by having different growth rates of economy and thus study how varying the growth rate of economy affects the influence of government expenditure on

inflation. Since government expenditure can contribute to both the growth rate of economy and rate of growth of prices, we can then conclude that the more rapid the country is developing, the less the rise of inflation rate would be as a result of government expenditure. As most of the government expenditure would be channeled to improve the growth of the economy rather than increase the prices (Edward Tower. 1971) [4]

- Preference:
  Our model assumes constant preference over allocation of time (working, shopping and leisure), thus by altering the elasticity of substitution among time allocation, we can study how varying preference over time allocation influence people's reaction to government policy such as tax, subsequently, the effectiveness of government policies on inflation. With the above two extensions, we might be able to progress our paper to a more thorough and complete level.

References

[1] William J Baumol and James Tobin. The optimal cash balance proposition: Maurice allais' priority. *Journal of Economic Literature*, 27(3):1160–62, 1989.

[2] Karl Brunner and Allan H Meltzer. Economies of scale in cash balances reconsidered. 1967.

[3] Thomas R Saving. Transactions costs and the demand for money. *The American Economic Review*, pages 407–420, 1971.

[4] Edward Tower. More on the welfare cost of inflationary finance: Comment. *Journal of Money, Credit and Banking*, 3(4):850–860, 1971.

[5] Sho-Chieh Tsiang. The role of money in trade-balance stability: Synthesis of the elasticity and absorption approaches. *The American Economic Review*, pages 912–936, 1961.