

Monetary Policy and Propensity to Hoarding

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Overview

- ▶ **Motivation**
- ▶ **Model Setup**
- ▶ **Steady State Analysis**
- ▶ **Simulation and Analysis**
- ▶ **Conclusion**

Motivation

- ▶ Drawbacks of Tsiang (1961)'s paper
 - Fails to explain the reason of money hoarding
 - The propensity to hoarding is constant
- ▶ Built on Saving (1971)'s framework
 - Money as a median of transaction
 - Build in a microfoundation of why individuals choose to hold money

Household

- ▶ Household Problem:

$$\sum_{t=0}^{\infty} \beta^t U(c_t, l_t) = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\rho}}{1-\rho} + \frac{l_t^{1-\eta}}{1-\eta} \right] \quad (1)$$

- ▶ Time Constraint:

$$l_t + n_t + s_t = l_t + n_t + \frac{c_t}{1 + m_{t+1}/p_t} \leq 1 \quad (2)$$

- ▶ Budget Constraint:

$$c_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} + k_{t+1} - (1-\delta)k_t \leq w_t n_t + r_t k_t + b_t - \tau_t + \frac{m_t}{p_t} + \Pi_t \quad (3)$$

Household: First Order Condition

- ▶ Leisure Choice:

$$[l_t] : \quad U_{l_t} = \mu_t \quad (4)$$

- ▶ Consumption Choice:

$$[c_t] : \quad \lambda_t = U_{c_t} - \mu_t s_{c_t} = U_{c_t} - U_{l_t} H_{c_t} \quad (5)$$

- ▶ Labor Supply Choice (labor supply function):

$$[n_t] : \quad \lambda_t w_t = \mu_t = U_{l_t} \quad (6)$$

Household: First Order Condition

- ▶ Capital Choice (capital supply function):

$$[k_{t+1}] : \quad \beta \lambda_{t+1} [r_{t+1} + 1 - \delta] = \lambda_t \quad (7)$$

- ▶ Bond Choice:

$$[b_{t+1}] : \quad \beta \lambda_{t+1} R_t = \lambda_t \quad (8)$$

- ▶ Money Holding:

$$[m_{t+1}] : \quad \beta \lambda_{t+1} \frac{1}{p_{t+1}} = \lambda_t \frac{1}{p_t} + \mu_t \frac{1}{p_t} H_{m_{t+1}/p_t} \quad (9)$$

Household: Money Demand Equation

$$\frac{R_t - Rm_t}{R_t} [U_{c_t} - U_{l_t} H_{c_t}] = -U_{l_t} H_{m_{t+1}}/p_t \quad (10)$$

Let $Rm_t = p_t/p_{t+1}$, real return on money holding, an inverse of inflation rate,

Firm

- ▶ Production Technology: $Y_t = A_t N_t^\alpha K_t^{1-\alpha}$
- ▶ Wage Rate (labor demand function):

$$w_t = \alpha A_t (N_t / K_t)^{\alpha-1} \quad (11)$$

- ▶ Capital Rental Rate (capital demand function):

$$r_t = (1 - \alpha) A_t (N_t / K_t)^\alpha \quad (12)$$

Government

- ▶ Government Budget Constraint:

$$g_t = \tau_t + \frac{B_{t+1}}{R_t} - B_t + \frac{M_{t+1} - M_t}{p_t} \quad (13)$$

Steady State Analysis

► : Analysis System

$$\begin{aligned}g - \tau - \left(\frac{1}{R} - 1\right)b &= \frac{m}{p}(1 - Rm) \\g - \tau_0 + b_0 &= \frac{m}{p} - \frac{m_0}{p_0} - \frac{b}{R} \\n + l + \left(1 + \frac{m}{p}\right)^{-1}c &= 1 \\c + \delta k + g &= An^{\alpha}k^{1-\alpha} \\(1 - \alpha)A\left(\frac{n}{k}\right)^{\alpha} &= R - 1 + \delta \\c^{-\rho}l^{\eta} - \left(1 + \frac{m}{p}\right)^{-1} &= \frac{1}{\alpha A}\left(\frac{n}{k}\right)^{1-\alpha} \\(c^{-\rho}l^{\eta} - \left(1 + \frac{m}{p}\right))\left(\frac{Rm}{R} - 1\right) &= -\left(1 + \frac{m}{p}\right)^{-2} \\R &= \beta^{-1}\end{aligned}\tag{14}$$

Steady State Analysis

- ▶ Money Demand Equation in Reduced Form:

$$\frac{m}{p} = f(Rm) = \left(\frac{\gamma^{1-\alpha}}{\alpha A}\right)^{-\frac{1}{2}} \left(1 - \frac{Rm}{R}\right)^{-\frac{1}{2}} - 1 \quad (15)$$

where $\gamma = \frac{n}{k} = \left[\frac{R-1+\delta}{(1-\alpha)A}\right]^{\frac{1}{\alpha}}$

- ▶ Future government budget constraint:

$$g - \tau + \frac{R-1}{R}b = f(Rm)(1 - Rm) \quad (16)$$

- ▶ Current government budget constraint:

$$m_0/p_0 = f(Rm) - (g - t_0 + b_0) + b/R \quad (17)$$

Steady State Analysis

- ▶ Initial Value: $g = 0.2$, $\tau = 0.15$, $\beta = 0.95$, $A = 1.3$,
 $b = 0.488$;
- ▶ the initial bond and money stock as: $b_0 = 3$ and $m_0 = 2$.

Table 1: Equilibrium Value

Variable	Situation 1)	Situation 2)
consumption c	0.4914	0.2579
labor input n	0.6843	0.4532
capital input k	0.7902	0.5233
leisure l	0.1746	0.3981
gross real interest rate R	1.0526	1.0526
real return on capital r	0.3526	0.3526
wage w	0.9501	0.9501
real money balance m/P	2.4800	0.7339
real return on money Rm	0.9700	0.7200
inflation rate π	0.0309	0.3889
bond issued b	0.4880	3.1100
seigniorage revenue $f(Rm)(1 - Rm)$	0.0744	0.2055
initial price level p_0	0.0231	11.6764

Steady State Analysis

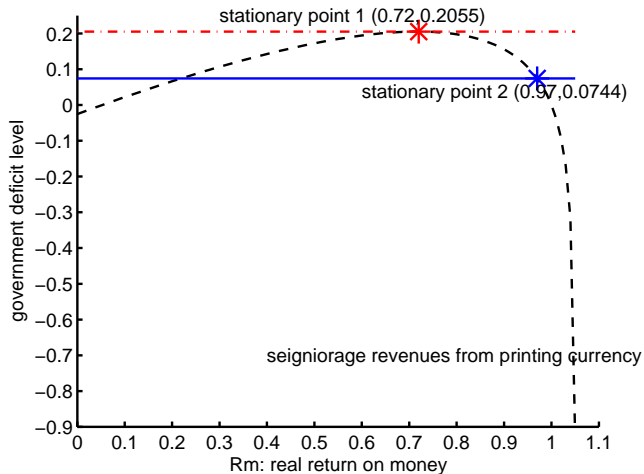


Figure: Determination of R_m

Simulation and Analysis: Differential Equations

$$\begin{aligned}\hat{w} &= \alpha A + (\alpha - 1)\hat{n} + (1 - \alpha)\hat{k} \\ \hat{r} &= \alpha \hat{A} + \alpha \hat{n} - \alpha \hat{k} \\ c^{-\rho} l^m [-\rho \hat{c} + \eta \hat{l}] &+ \frac{m}{\rho} \left(1 + \frac{m}{\rho}\right)^{-2} \left(\frac{\hat{m}}{\rho}\right) \\ &= \frac{\eta}{\alpha A} \left(\frac{n}{k}\right)^{1-\alpha} [-\hat{A} + (1 - \alpha)\hat{n} - (1 - \alpha)\hat{k}] \\ &\quad - \hat{A} + (1 - \alpha)\hat{n} - (1 - \alpha)\hat{k} + \frac{Rm}{Rm - R} (\hat{R}m - \hat{R}) \\ &= -2 \left(1 + \frac{m}{\rho}\right)^{-1} \frac{m}{\rho} \left(\frac{\hat{m}}{\rho}\right)\end{aligned}\tag{19}$$

Simulation and Analysis: Differential Equations

$$g\hat{g} - \tau\hat{\tau} + \frac{b}{R}\hat{R} - (\frac{1}{R} - 1)b\hat{b} = \frac{m}{p}(1 - Rm)(\frac{\hat{m}}{p}) - \frac{m}{p}Rm\hat{R}m$$

$$g\hat{g} - \tau_0\hat{\tau}_0 + b_0\hat{b}_0 = \frac{m}{p}(\frac{\hat{m}}{p}) - \frac{m_0}{p_0}(\hat{m}_0 - \hat{p}_0) - \frac{b}{R}(\hat{b} - \hat{R})$$

$$n\hat{n} + \hat{l}l + (1 + \frac{m}{p})^{-1}c\hat{c} - (1 + \frac{m}{p})^{-2}c\frac{m}{p}(\frac{\hat{m}}{p}) = 0$$

$$c\hat{c} + \delta k\hat{k} + g\hat{g} = An^\alpha k^{1-\alpha}[\alpha\hat{n} + (1 - \alpha)\hat{k} + \hat{A}]$$

$$\alpha\hat{n} + \hat{A} - \alpha\hat{k} = R\hat{R}$$

(18)

Table 2: Transition Matrix Around Low Inflation Rate ($Rm = 0.97$)

Variable	\hat{A}	\hat{g}	\hat{t}	\hat{t}_0	\hat{b}	\hat{b}_0	\hat{m}_0	R
c	1.9269	-0.6179	-0.0249	0.0000	0.0041	0.0000	0.0000	0.1235
n	-0.3508	-0.0897	-0.0160	0.0000	0.0026	0.0000	0.0000	0.2491
k	3.3555	-0.6179	-0.0249	0.0000	0.0041	0.0000	0.0000	-1.3802
l	1.3943	-0.2264	0.8914	0.0000	-0.1450	0.0000	0.0000	-2.4992
r	-1.5944	0.3698	0.0062	0.0000	-0.0010	0.0000	0.0000	1.1405
w	2.1119	-0.1585	-0.0027	0.0000	0.0004	0.0000	0.0000	-0.4888
m/P	1.3449	-0.9188	0.6891	0.0000	-0.1121	0.0000	0.0000	-1.2127
Rm	0.0416	-0.1116	0.0837	0.0000	-0.0136	0.0000	0.0000	-0.2302
p_0	-0.0386	0.0287	-0.0198	-0.0016	0.0086	0.0347	1.0000	0.0294

Simulation and Analysis: Policy Changes

- ▶ Changes in government expenditure
- ▶ Changes in lump sum tax
- ▶ Changes in bond issuing

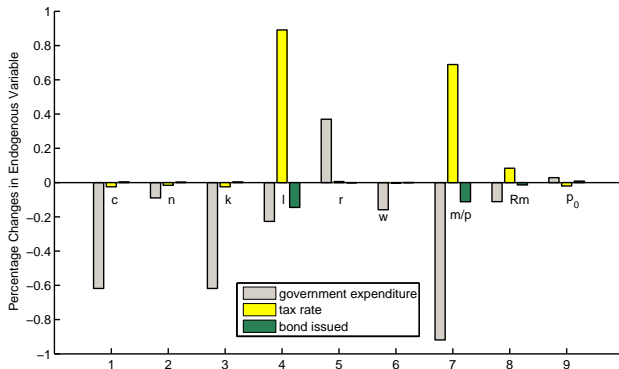


Figure: Percentage Changes in Government Expenditure, Tax Rate and Bond Issued

Simulation and Analysis: External Shocks

- ▶ Technology Shocks
- ▶ Interest Rate Changes

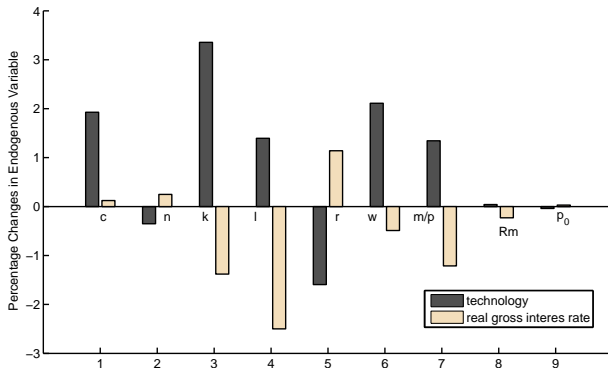


Figure: Percentage Changes in Technology and Real Gross Interest Rate

Extension

- ▶ Include economic growth (Tower, 1971)
- ▶ Changes in preference
- ▶ Two-country two-goods economy