



THE OPTIMUM MINIMUM WAGE WHEN LABOR SERVICES ARE TAXED

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Introduction

Our model is based on three previous works in minimum wage research. In those models, an economy (e.g. Chile 1973-1983) is considered with a minimum wage sector and a free sector, and a tax imposed on labor earnings. Supply on labor is positively related to wage and jobs in minimum wage sector are allocated by lottery.

Edwards and Edwards (1991) find that workers prefer jobs in minimum wage sector but they still accept job in the free sector as long as wages in free sector exceed their reservation wage. They view workers with reservation wage less than the minimum wage sector but higher than the free sector as unemployed.

Gang and Tower (1990) introduce the employment lottery, under which imposition of minimum wage in one sector may increase aggregate employment. In addition, they discover that though minimum wage can raise employment, it will reduce economics welfare when the utility of leisure is accounted for.

Gokcekus and Tower (1996) explore in the paper “An Efficiency Enhancing Minimum Wage” of conditions for an increase of minimum wage 1) to boost employment, and 2) to raise economic efficiency if labor services are taxed. They conclude that such condition for both situations is elasticity of demand for labor in the free sector exceeds that of the minimum wage sector.

In this project, we mainly focus on finding the optimum wage, and how much employment and economic welfare it creates. Here we use GAMS to simulate in and the process is displayed in the appendix.

STRUCTURE OF THE MODEL

1. Assumptions:

- 1) There are 2 sectors in economy, manufacturing (M) and food (F).
- 2) Capital and labor produce manufacturing and Labor and Land produce food.
- 3) M and F are traded freely at fixed world price.
- 4) Production functions are Cobb-Douglas and initial output shares in domestic product are equal.
- 5) Exchange rate is fixed that nominal wages equal real wages, and labor is mobile between two sectors.
- 6) Minimum wage in manufacturing sector and flexible wage rate in food sector. Initially minimum wage equals free wage.
- 7) Output of both sectors is taxed at ad valorem tax rate t and all tax revenues are rebated to consumers.

Since output in both sectors are determined by labor and another fixed factor, the formula is equivalent to only labor earnings as tax on the fixed factor has no impact apart from income distribution.

2. Mechanisms

As there is minimum wage in manufacturing sector reduce employment and output, it drives workers to the free sector thus raising employment and output while depressing wage in food sector. The additive aggregate utility function is as follows:

utility = consumption + leisure

The consumption is expenditure on both manufacture goods and food, and leisure is the utility of increased leisure from minimum wage.

In the simulation, we measure leisure as the integral of the area under the supply of labor. For worker attracted into the manufacturing sector due to higher wage, leisure is negative; and for worker repelled by the lower food wage leisure is positive.

STRUCTURE OF THE MODEL

3. Variables and parameters

Variables	
l_m	labor employed in manufacture
l_f	labor employed in food
L	total labor employed
w_m	wage rate in manufacture
w_f	wage rate in food
M	production in manufacture
F	production in food
A_m	applicants for manufacture
A_f	applicants for food
W_{in}	proportion of lottery winners
L_{ose}	proportion of lottery losers
Con	consumption level
Le_i	leisure level
W_{elf}	welfare level
dM	the percent by which the minimum wage exceeds the free wage
dW	the percent change in welfare
dE	the percent change in employment
dY	the percent change in output

4. Equations

Equations	
production function in Food	$F=B*(T**(1-Beta))*(L_f**Beta);$
production function in Manufacture	$M=A*(K**(1-Alpha))*(L_m**Alpha);$
wage rate in manufacture	$W_m=Alpha*A*((K/L_m)**(1-Alpha));$
wage rate in food	$W_f=Beta*B*((T/L_f)**(1-Beta));$
total labor employment	$L=L_f+L_m;$
labor employment in food	$A_m=C*(W_m**s);$
applicants for manufacture	$A_f=C*(W_f**s);$
applicants for food	$W_{in}=L_m/A_m;$
lottery winners ratio	$L_{lose}=1-W_{in};$
lottery losers ratio	$L_f=L_{lose}*A_f;$
consumption level	$Con=(1+tax)*(P_f*F+P_m*M);$
Leisure level	$Lei=(s*C*(1-W_{in}*W_m**(s+1)-L_{lose}*W_f**(s+1)))/(s+1);$
Welfare level	$Welf=Con+Lei;$
percent wage difference	$dM=(W_m-W_f)/W_f;$
percent welfare change	$dW=(Welf-Welf_0)/Welf_0;$
percent employment change	$dE=(L-L_0)/L_0;$
percent output change	$dY=(M+F-M_0-F_0)/(M_0+F_0);$

5. Initialization and Normalization

Table 2 Initial and Normalization of 5 Cases

	A	B	C	K	T
Case1(minimum wage): Alpha=0.5Beta=0.6 S=1Tax=1	2	5/3	11	5	6
Case2(minimum wage2): Alpha=0.2Beta=0.6 S=1Tax=1	5	5/3	4	1	3
Case3(minimum wage3): Alpha=0.1Beta=0.9 S=1Tax=1	10	10/9	10	1	9
Case4(minimum wage4): Alpha=0.5Beta=0.5 S=1Tax=1	2	2	2	1	1
Case5(minimum wage5): Alpha=0.5Beta=0.6 S=0Tax=1	2	5/3	11	5	6

We name alpha as labor's share in food sector, and beta as labor's share in manufacture sector. S is the labor supply elasticity, and we make technology in manufacture to be A , technology in food be B and shift in labor supply be C . K is capital in manufacture and T represents land in food. The initial values we set are presented in Table 1 below.

There are 5 Gam files in total for five cases. In each case, since we want to normalize the initial wage in manufacture and food to 1 ($W_{mo}=W_{fo}$) and the production share of manufacture and food equal to each other ($M_o=F_o$), we adjust the parameter of A, B, C and exogenous variables K and T . In the following table1 we list the assignment.

To develop a sense of how large the optimum minimum wage might be, and how important its effects are, we simulate the effect of labor supply elasticity and relative labor demand elasticity of the two sectors. In our case of the Cobb-Douglas production, the labor demand elasticity can be calculated by the labor share (Labor demand elasticity = $1 / (1 - \text{labor share})$). Two extreme case of labor supply elasticity are considered ($s=1$ and $s=0$). Under the sufficiently high labor supply elasticity, we compare 4 cases where the difference between labor demand elasticity in two sectors enlarges from 0 to 9.

SIMULATION RESULT & CONCLUSION

Table 2 The effect of minimum wage on welfare, employment and output

Tax as a proportion of producer price				1	
Labor's shares (alpha/beta) & demand elasticities (δ) in M & F		Labor supply elasticity	%		
Alpha=0.5 $\delta_m=2$	Beta=0.6 $\delta_f=2.5$	1	dm dw de dy	2.020 0.010 0.065 0.030	
Alpha=0.2 $\delta_m=1.25$	Beta=0.6 $\delta_f=2.5$	1	dm dw de dy	7.356 0.051 0.427 0.145	
Alpha=0.1 $\delta_m=1.111$	Beta=0.9 $\delta_f=10$	1	dm dw de dy	17.173 0.148 0.955 0.412	
Alpha=0.5 $\delta_m=2$	Beta=0.5 $\delta_f=2$	1	dm dw de dy	0 0 0 0	
Alpha=0.5 $\delta_m=2$	Beta=0.6 $\delta_f=2.5$	0	dm dw de dy	0 0 0 0	

Table 2 displays our results. The simulations are carried out using the General Algebraic Modeling System (GAMS) and the entries in each cell are explained in the legend below the table. We used Cobb-Douglas production function, so labor shares are constant. δ_i is the demand elasticity of sector i , which in this case is manufacturing or food. It is demonstrated as $\delta=1/[1-\Theta_i]$, and Θ is the labor shares which is alpha or beta here.

As can be seen in the simulation result, the most favorable situation for a minimum wage is that when elasticity of demand for manufacturing is merely 1.111 while for food amounts to 10 and tax imposed on output is 100%. In this case, the optimum wage goes 17%, highly above the free wage and welfare increases slightly at 0.15%, employment rises 1% and output grows by 0.4%. As for other cases, the percentage wage difference (dm) due to optimum minimum wage declines as the difference of labor demand elasticity between two sectors shrinks.

For case 4 and 5, we want to demonstrate that when the labor demand elasticity ($1/(1-\alpha))= (1/(1-\beta))$) are the same in two sectors or when the labor supply elasticity is zero, the optimal minimum wage in the covered sector is just equal to the wage in the uncovered sector.

Developing an economy with a tax on all labor earnings, we discovered that a slightly binding minimum wage on one sector can improve efficiency. Minimum wage may increase employment and output by drawing additional workers into employment. Efficiency and tax revenue rises as the minimum wage drives labor out of untaxed leisure, where too much of the labor force is lurking into taxed work. It can be concluded that a slight binding minimum wage can raise tax revenue, employment rate and economic efficiency if and only if the elasticity of demand in free sector exceeds that in minimum wage sector.

From this simulation, we discovered that the optimum minimum wage is nonbinding under the undistorted economy, which is the case presented in GT's paper. Additionally, Gokcekus and Tower (1996) find analytically that an incremental increase in the minimum wage in either sector, starting from the free market level will not change welfare if the labor demand elasticities are equal or if the labor supply elasticity is zero.

The reason is, in this cases employment is not changed by an infinitesimal change in the minimum wage, so the free labor market is a welfare maximum. These results are confirmed by our simulations.

Is minimum wage beneficial or harmful? We cannot give a definite answer because either models showing the benefits or damages in and can be easily draw up by economist. Generally, a fixed distortion in one market will call for a non-zero distortion in another market, which is a result of this paper.

APPENDIX

*Efficiency Enhancing Minimum Wage Model

Parameters

Alpha labor share in manufacture

Beta labor share in food

A technology in manufacture

B technology in food

labor supply elasticity

C shift in labor supply

K capital in manufacture

T land in food

Pf food price

Pm manufacture price

tax tax rate

Wfo initial wage rate in food

Wm initial wage rate in manufacture

Lm initial labor in manufacture

Lfo initial labor in food

Lo initial labor supply

M_0 initial production of manufacture

F_0 initial production of food

A_{m0} initial applicants in manufacture

A_{f0} initial applicants in food

W_{i0} initial winning rate

L_{o0} initial losing rare

L_{e0} initial leisure level

C_{n0} initial consumption level

W_{e0} initial welfare level

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*Initialization & Normalization

*Directly Asssigned

$\alpha=0.5$;

$\beta=0.6$;

$A=2$;

$B=(5/3)$;

$s=1$;

$C=11$;

$K=5$;

$T=6$;

$P_f=1$;

$$Pm=1;$$

$$tax=1;$$

$$Wfo=1;$$

$$Wmo=1;$$

*Indirectly Calculated

$$Amo= C*Wmo**s;$$

$$Afo= C*Wfo**s;$$

$$Lmo= K/((Wmo/(Alpha*A))**(1/(1-Alpha)));$$

$$Lfo= T/((Wfo/(Beta*B))**(1/(1-Beta)));$$

$$Lo= Lmo+Lfo;$$

$$Mo= A*(K**(1-Alpha))*(Lmo**Alpha);$$

$$Fo= B*(T**(1-Beta))*(Lfo**Beta);$$

$$Wino= Lmo/Amo;$$

$$Loseo= 1-Wino;$$

$$Cono= (1+tax)*(Pf*Fo+Pm*Mo);$$

$$Leio= (s*C*(1-Wino*Wmo**(s+1)-Loseo*Wfo**(s+1)))/(s+1);$$

$$Welfo= Cono+Leio;$$

Display Alpha, Beta, A, B, s, C, K, T, Pf, Pm, tax, Wfo, Wmo, Amo, Afo, Lmo, Lfo, Lo,
Mo, Fo, Wino, Loseo, Cono, Leio, Welfo;

Variables

L_m labor employed in manufacture

L_f labor employed in food

L total labor employed

W_m wage rate in manufacture

W_f wage rate in food

M production in manufacture

F production in food

A_m applicants for manufacture

A_f applicants for food

W_{in} proportion of lottery winners

L_{lose} proportion of lottery losers

Con consumption level

Le_i leisure level

$Welf$ welfare level

dM the percent by which the minimum wage exceeds the free wage

dW the percent change in welfare

dE the percent change in employment

dY the percent change in output

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$Lm.L = Lmo;$

$Lf.L = Lfo;$

$L.L = Lo;$

$Wm.L = Wmo;$

$Wf.L = Wfo;$

$M.L = Mo;$

$F.L = Fo;$

$Am.L = Amo;$

$Af.L = Afo;$

$Win.L = Wino;$

$Lose.L = Loseo;$

$Con.L = Cono;$

$Lei.L = Leio;$

$Welf.L = Welfo;$

$dM.L = 0;$

$dW.L = 0;$

$dE.L = 0;$

$dY.L = 0;$

Equations

Manufacture production function in Manufacture

Food production function in Food

WageM wage rate in manufacture

WageF wage rate in food

Labor total labor employment

LaborF labor employment in food

ApplicationM applicants for manufacture

ApplicationF applicants for food

Winr lottery winners ratio

Loser lottery losers ratio

Consumption consumption level

Leisure Leisure level

Welfare Welfare level

dwage percent wage change due to minimum wage

dwelfare percent welfare change

deployment percent employment change

doutput percent output change

;

Food.. $F=e*B*(T**(1-Beta))*(Lf**Beta);$

$$\text{Manufacture.. } M=e=A*(K**(1-\text{Alpha}))*(\text{Lm}**\text{Alpha});$$

$$\text{WageM.. } Wm=e=\text{Alpha}*A*((K/\text{Lm})**(1-\text{Alpha}));$$

$$\text{WageF.. } Wf=e=\text{Beta}*B*((T/\text{Lf})**(1-\text{Beta}));$$

$$\text{Labor.. } L=e=\text{Lf}+\text{Lm};$$

$$\text{ApplicationM.. } Am=e=C*(Wm**s);$$

$$\text{ApplicationF.. } Af=e=C*(Wf**s);$$

$$\text{Winr.. } Win=e=\text{Lm}/\text{Am};$$

$$\text{Loser.. } Lose=e=1-\text{Win};$$

$$\text{LaborF.. } \text{Lf}=e=Lose*\text{Af};$$

$$\text{Consumption.. } \text{Con}=e=(1+\text{tax})*(P_f*\text{F}+P_m*\text{M});$$

$$\text{Leisure.. } \text{Lei}=e=(s*C*(1-\text{Win}*Wm**(s+1)-Lose*Wf**(s+1)))/(s+1);$$

$$\text{Welfare.. } \text{Welf}=e=\text{Con}+\text{Lei};$$

$$\text{dwage.. } dM=e=(Wm-Wf)/Wf;$$

$$\text{dwelfare.. } dW=e=(\text{Welf}-\text{Welfo})/\text{Welfo};$$

$$\text{deployment.. } dE=e=(L-\text{Lo})/\text{Lo};$$

$$\text{doutput.. } dY=e=(M+\text{F}-\text{Mo}-\text{Fo})/(\text{Mo}+\text{Fo});$$

Model Wage /All/;

Solve Wage using NLP maximizing Welf;

REFERENCES

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