New German Transfer Problem

Class: Econ 567

Quanhe Wang & Shi Jing

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Project for Computer Modeling

Quanhe

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1. Abstract

The German transfer problem arose in the late 1920's where the victorious Allies demanded large reparations from Germany after WWI. The payment problem can be divided into two parts: the first problem is Budgetary where extracting the German's money from their pockets and paying them to the Agent-General and the second problem is transfer problem is converting German money directly to France. The issue of how international transfers affect the terms of trade was raised in a famous debate between two great economists, Bertil Ohlin (one of the originators of the factor proportions theory of trade) and John Maynard Keynes. Hence, an analysis of international transfer is also useful in understanding how it affects terms of trade.

2. Introduction

This is a simplified model which involves two countries (France and Germany), two commodities where one of them is tradable and the other one is not. By setting different initial endowment labor and leisure time, we get different German indifference curves after transfer. Moreover, we assume the model takes place under full employment, with all the wage received contributing to domestic consumption and labor is the only factor of production. German people could choose work or not work (leisure) and their utility depends on a bundle of consumption and leisure. Suppose now German makes reparation to France, they will work harder than before to finance the transfer. Therefore, we developed two simulations to analyze the effects of German reparations on German economy as follows:

Simulation 1: the German government financed reparations by lump sum tax.

Simulation 2: the German government financed reparations by ad valorem tax.

3. Models and Simulations

3.1 Simulation 1 Financing reparations by lump sum tax

3.1.1 Variables:

In order to debug this model, we generated several new variables and equations such as normalized utility and full income:

<u>Exogenous variables</u>

e – Efficiency of German labor time

R – Reparations paid from Germany to France

<u>Endogenous variables</u>

Le – Leisure time

L – Labor time

Y – Production of toys

w – Real wage rate

U – Utility

CO – Initial consumption

3.1.2 Basic Equations

1) **Utility function**

   $U = Le^{\alpha} * C^{1-\alpha}$($\alpha$ = $Le/(Le+CO)$, is the share of leisure in normalized utility or fullincome)

   $UO = Le^{\alpha} * CO^{1-\alpha}$

2) **Time Balance**

   $Le + L = 100$

3) **Income Function**

   $Y = e * L$

4) **Reparation Function**

   $CO = Y\text{-}RO$($Sigma = CO/(CO+RO),$ is the share of consumption in output Y)

5) **Fullincome Function**

   $fullincome = Le+CO$

   $UUO = fullincome / UO$

$Unormalized = UUO * UO$

6) **Wage Function**

$$\frac{dU/dC}{dU/dLe} = w$$

PS: In seeking to maximize utility, the individual is bound by two constraints,

$$Le + L = 100 \text{ and } Y = wL$$

Setting up the Lagrangian function:

$$L = U(Le, C) + \lambda(w - C - wLe)$$

The first order conditions:

$$\frac{dL}{dC} = \frac{dU}{dC} - \lambda = 0$$
$$\frac{dL}{dLe} = \frac{dU}{dL} - \lambda = 0$$

Dividing the two, we get

$$\frac{dU/dLe}{dU/dC} = w = MRS(Le \text{ for } C)$$

That means, in order to maximize utility, the individual should choose to work that number of hours for which the $MRS(of\ Le$ for $C)$ is equal to w.

3.1.3 Proportional change in the above equations:

1) **Utility function**

$U^\wedge = \alpha Le^\wedge + \alpha C^\wedge$

2) **Time Balance**

$Le * Le^\wedge + L * L^\wedge = 0$

3) **Income Function**

$Y^\wedge = w^\wedge + L^\wedge$

4) **Reparation Function**

$Y^\wedge - 0.2R^\wedge = 0.8C^\wedge$

5) **Wage Function**

$Le^\wedge = C^\wedge + w^\wedge$

3.1.4 Policy matrix and Results:

- **Free parameters:**

Le – leisure time

L – Labor time

e - Working efficiency

w – Wage rate

RO – reparations

- **Other parameters:**

  Y - Output

  CO - Initial Consumption

  Alpha - leisure share in fullincome

  Sigma - share of consumption in output

  UUO – fullincome/UO

  UO – Initial Utility

  Fullincome/Unormalized – *Le+CO*

PS: We debugged our simulation 1 model by using:

d(Unormalized) / dC = d(Unormalized) / dC = (1-α)*UUO*(Le^α)*CO^(-α) = 1.

Next, we observed the impact of reparations R on endogenous variables, and debugged our model: Plugging other parameters into: dUnormalized/dC= (1-alpha)*UUO*(Le^alpha)*CO^(-alpha) = 1, because one unit increase in reparation leads to an equivalent amount of change in normalized utility with a negative sign. Then, we compared outcomes derived from 3 sets of parameters as follow:

Set 1 when Le=80, L=20

Free parameters		Other parameters	
Le	80.000	Y	20.000
L	20.000	CO	10.000
e	1.000	alpha	0.889
w	1.000	sigma	0.500
RO	10.000	UUO	1.417
		UO	63.496
		fullincome	90.0000
		Unormalized	90.0000

Matrix of system of equations:

Matrix	Unormalized^	Le^	C^	L^	y^	w^	equals	R^	e^
1	1.0000	-0.1111	-0.8889	0.0000	0.0000	0.0000		0.0000	0.0000
2	0.0000	10.0000	0.0000	90.0000	0.0000	0.0000		0.0000	0.0000
3	0.0000	0.0000	0.0000	-1.0000	1.0000	0.0000		0.0000	1.0000
4	0.0000	0.0000	0.8889	0.0000	-1.0000	0.0000		-0.1111	0.0000
5	0.0000	1.0000	-1.0000	0.0000	0.0000	-1.0000		0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000		0.0000	1.0000

Policy matrix:

	R^	e^
U^	-0.111111111	0.222222
Le^	-0.111111111	0.111111
C^	-0.111111111	1.111111
L^	0.444444444	-0.44444
y^	0.444444444	0.555556
W^	0	1

Explanations:

As indicated in the above matrix, Germans started with working less, while the reparation makes them worked harder than before. When German makes a transfer to France, their utility, leisure time and consumption go down. Consequently, their labor time and production of goods expand. However, the function of market efficiency has different effect than that of reparations. On one hand, when German market becomes more efficient, their utility, leisure time, output and consumption go up. On the other hand, German labor time decreases because their working efficiency increases. As a result, lump sum tax has a larger effect on consumption and production since this tax is a fixed amount and is not subject to anything taxpayers can change.

Set 2 when Le=50, L=50

Free parameters		Other parameters	
Le	50.000	Y	50.000
L	50.000	C0	40.000
e	1.000	alpha	0.556
w	1.000	sigma	0.800
R0	10.000	UU0	1.988
		U0	45.279
		fullincome	90.0000
		Unormalized	90.0000

Matrix of system of equations:

Matrix	Unormalizedˆ	Leˆ	Cˆ	Lˆ	yˆ	wˆ	equals	Rˆ	eˆ
1	1	-0.55556	-0.44444	0	0	0		0	0
2	0	50	0	50	0	0		0	0
3	0	0	0	-1	1	0		0	1
4	0	0	0.8	0	-1	0		-0.2	0
5	0	1	-1	0	0	-1		0	0
6	0	0	0	0	0	1		0	1

Policy matrix:

	R^	e^
U^	-0.111111111	0.555556
Le^	-0.111111111	0.111111
C^	-0.111111111	1.111111
L^	0.111111111	-0.11111
y^	0.111111111	0.888889
W^	0	1

Explanation:

As can be seen in the above matrix, this is a special case where Germans spend their labor time and leisure time evenly. In this case, when Germany makes a transfer to France with the same amount in set 1, their labor time and output increase less than that of set 1 because they work harder initially. With respect to economic efficiency, the utility increases more in this case than in previous case. However, proportional changes in leisure time and consumption do not depend on the initial endowment of themselves.

Set 3 when Le=20, L=80

Free parameters		Other parameters	
Le	20.000	Y	80.000
L	80.000	C0	70.000
e	1.000	alpha	0.222
w	1.000	sigma	0.875
R0	10.000	UU0	1.698
		U0	52.990
		fullincome	90.0000
		Unormalized	90.0000

Matrix of system of equations:

Matrix	Unormalizedˆ	Leˆ	Cˆ	Lˆ	yˆ	wˆ	equals	Rˆ	eˆ
1	1	-0.22222	-0.77778	0	0	0		0	0
2	0	20	0	80	0	0		0	0
3	0	0	0	-1	1	0		0	1
4	0	0	0.875	0	-1	0		-0.125	0
5	0	1	-1	0	0	-1		0	0
6	0	0	0	0	0	1		0	1

Policy matrix:

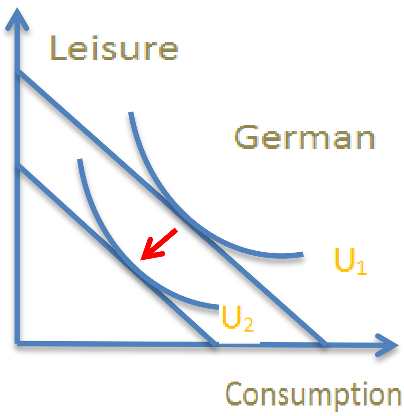
	R^	e^
U^	-0.111111111	0.888889
Le^	-0.111111111	0.111111
C^	-0.111111111	1.111111
L^	0.027777778	-0.02778
y^	0.027777778	0.972222
W^	0	1

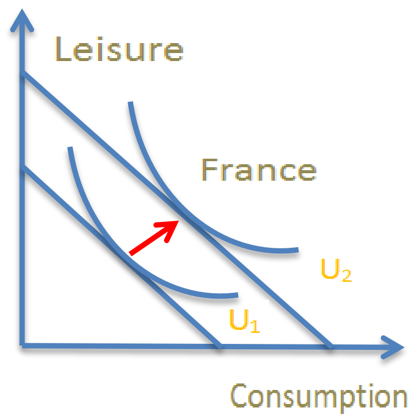
Explanation:

Let's focus on case 3 where Germans start with working harder. In this case, when Germany makes a transfer to France with the same amount as usual, reparation has a tiny small effect on labor time and production. With respect to economic efficiency, the utility and output increase more in this case than in previous case because labor occupies a large amount in Germans lifetime. However, reparation does not affect wages since wage depends on working efficiency solely.

Summarized Result:

In all these three cases, utility, leisure and consumption would decrease in the same proportion with one percent increase in reparation. No matter how much leisure time accounts in German's lifetime, the reparation does not affect wages, leading to same proportional change in labor and output. Additionally, given a transfer from Germany to France will lower the German utility, hence the budget constraint of Germans will shift inward while the budget constraint of Frenchmen will shift outward. This shift is parallel as wage is the slope of budget constraint, but it does not change with reparations. A graph as follow could better illustrate this scenario more straightforward:





3.2 Simulation 2 Financing reparations by ad valorem tax

3.2.1 Variables:

<u>Exogenous variables</u>

e – Efficiency of German labor time

R – Reparations paid from Germany to France

<u>Endogenous variables</u>

Le – Leisure time

L – Labor time

Y – Production of toys

wr – Real wage received

U – Utility

CO – Initial consumption

t – Tax rate

3.2.2 Basic Equations

1) **Utility function**

$U = Le^\wedge \alpha C^\wedge(1-\alpha)$

$UO = Le^\wedge \alpha * CO^\wedge(1-\alpha)$

2) **Time Balance**

$Le + L = 100$

3) **Income Function**

$Y = wL$

4) **Reparation Function**

$Y - R = C$

$R = TR(tax\ revenue)$

5) **Wage Function**

$wr = e(1-t)$

$$\frac{dU/dC}{dU/dLe} = wr$$

6) **Fullincome Function**

fullincome = Le+CO

UUO = fullincome / UO

*Unormalized = UUO * UO*

7) **Tax Revenue Function**

$TR = e * L * t$

3.2.3 Proportional change in the above equations:

1) **Utility function**

$U^\wedge = \alpha Le^\wedge + \alpha C^\wedge$

2) **Time Balance**

$Le * Le^\wedge + L * L^\wedge = 0$

3) **Income Function**

$Y^\wedge = e^\wedge + L^\wedge$

4) **Reparation Function**

$R^\wedge = Y^\wedge - R^\wedge$

5) **Wage Function**

$$Le^{\wedge} = C^{\wedge} + wr^{\wedge}$$

6) **Tax Function**

$$R^{\wedge} = Y^{\wedge} + dt$$

3.2.4 Policy matrices and Results:

Free parameters:

Le – leisure time

L – Labor time

e – Working efficiency

RO – reparations

Other parameters:

t – tax rate

Y - Output

CO - Initial Consumption

Alpha - leisure share in fullincome

Sigma - share of consumption in output

UUO - fullincome/UO

UO - Initial utility

wr – wage received

Fullincome/Unormalized – *Le*+*CO*

Next, we observed the impact of reparations R on endogenous variables, and compared outcomes derived from 3 sets of parameters as follow:

Set 1 when Le=80, L=20

Free parameters		Other parameters	
Le	80.000	t	0.5
L	20.000	Y	20
E	1.000	CO	10
RO	10.000	alpha	0.8
		UUO	0.947323
		UO	52.78032
		fullincome	50
		Unomarlized	50
		wr	0.5

Matrix of system of equations:

matrix	Unormalized^	Le^	C^	L^	Y^	wr^	dt	equals	e^	R^
1	1	-0.8	-0.2	0	0	0	0		0	0
2	0	80	0	20	0	0	0		0	0
3	0	0	0	-1	1	0	0		1	0
4	0	0	0	0	0	1	1		1	0
5	0	-1	1	0	0	-1	0		0	0
6	0	0	0	0	0.5	0	1		0	0.5
7	0	0	-0.5	0	1	0	0		0	0.5

Policy matrix:

	e^	R^
Unormalized^	0.342857	-0.14286
Le^	0.0714	-0.07143
C^	1.4286	-0.42857
L^	-0.2857	0.285714
Y^	0.7143	0.285714
wr^	1.3571	-0.35714
dt	-0.3571	0.357143

Set 2 when Le=50, L=50

Free parameters		Other parameters	
Le	50.000	t	0.2
L	50.000	Y	50
e	1.000	CO	40
RO	10.000	alpha	0.5
		UUO	1.788854
		UO	44.72136
		fullincome	80
		Unomarlized	80
		wr	0.8

Matrix of system of equations:

matrix	Unormalized^	Le^	C^	L^	Y^	wr^	dt	equals	e^	R^
1	1	-0.5	-0.5	0	0	0	0		0	0
2	0	50	0	50	0	0	0		0	0
3	0	0	0	-1	1	0	0		1	0
4	0	0	0	0	0	1	1		1	0
5	0	-1	1	0	0	-1	0		0	0
6	0	0	0	0	0.2	0	1		0	0.2
7	0	0	-0.8	0	1	0	0		0	0.2

Policy matrix:

	e^	R^
Unormalized^	0.621951	-0.12195
Le^	0.0244	-0.02439
C^	1.2195	-0.21951
L^	-0.0244	0.02439
Y^	0.9756	0.02439
wr^	1.1951	-0.19512
dt	-0.1951	0.195122

Set 3 when Le=20, L=80

Free parameters		Other parameters	
Le	20.000	t	0.125
L	80.000	Y	80
e	1.000	CO	70
RO	10.000	alpha	0.2
		UUO	1.605919
		UO	54.48594
		fullincome	87.5
		Unomarlized	87.5
		wr	0.875

Matrix of system of equations:

matrix	Unormalized^	Le^	C^	L^	Y^	wr^	dt	equals	e^	R^
1	1	-0.2	-0.8	0	0	0	0		0	0
2	0	20	0	80	0	0	0		0	0
3	0	0	0	-1	1	0	0		1	0
4	0	0	0	0	0	1	1		1	0
5	0	-1	1	0	0	-1	0		0	0
6	0	0	0	0	0.125	0	1		0	0.125
7	0	0	-0.875	0	1	0	0		0	0.125

Policy matrix:

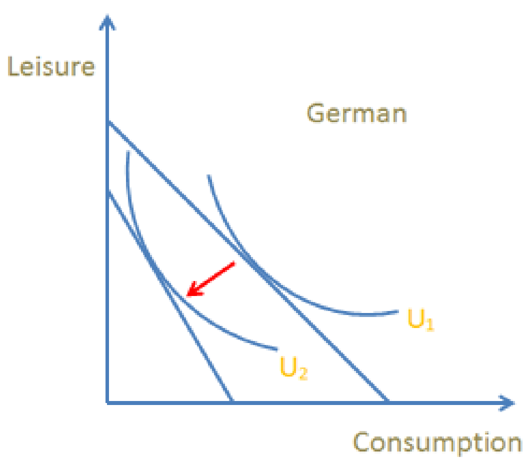
	e^	R^
Unormalized^	0.913879	-0.11388
Le^	0.0142	-0.01423
C^	1.1388	-0.13879
L^	-0.0036	0.003559
Y^	0.9964	0.003559
wr^	1.1246	-0.12456
dt	-0.1246	0.124555

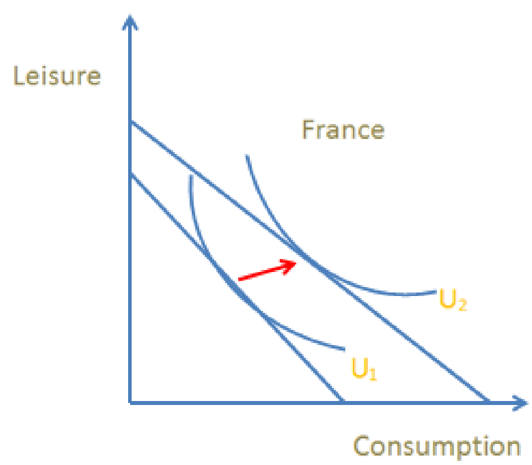
4. Interpretation of the Results

As can be seen, when market becomes more efficient, German will have more utility, leisure and output. Moreover, your consumption increases much higher than leisure because both your wage and leisure increase.

When German makes a transfer to France, they will end up with working harder as well as their utility goes down and consume less. Since they work harder than before, they will have the same proportion increase in both labor and output. Additionally, wage rate decreases with the same rate as tax increases.

You may wonder why our result is right: We check it right here. From the two graphs, we can see: Given a transfer to France, German utility will shift inward while France utility will shift outward. However, the budget constraint won't shrink parallel as can be seen from the following graph, because the impact of consumption caused by reparation is larger than the impact of leisure. In these two simulations, we simply changed the initial endowments where we get different results. But, anyway, the trend of endogenous variables is unchanged.





If the country produces two goods, then a tax on one good would have the same effect on subsidizing the other one. Therefore, if an ad valorem tax was imposed on labor would drive some German out of labor market to consume more leisure. Since the same proportion of leisure increase and wage received decrease cancels out, the consumption will not change. Tax rate would not influence tax revenue because the amount of reparations is initially required by French government.

An increase in reparations has no effect on wage received because the transfer was financed by collecting ad valorem tax where would not affect real wage received.

An increase in efficiency impacts only the labor and leisure sector, decreasing labor and increasing leisure proportionately. There are no changes in consumption, in come and tax revenue because we move along the leisure-demand curve – there is no shift. With respect to tax rate, one percentage tax will increase leisure and utility because more people would give up working harder by government tax collection.

5. Conclusion

When we have tried different value of leisure and labor, we found that different effects influenced by reparations. Say if Germans initially spend more time in leisure, in other words, they used to be very lazy, now they will suffer a larger lost in leisure and consumption. But on the other hand, if Germans initially spend more time in labor, in other words, they used to be very industrious, now they will suffer smaller lost in leisure and consumption. These discrepancies are maybe part of different assumptions in debate.

In general, a transfer worsens the donor's terms of trade if the donor has a higher marginal propensity to spend on its export than the recipient. If the donor has a lower marginal propensity to spend on its export, its terms of trade will actually improve.

6. Appendix

Simulation 1:

Free parameters:

Le	80.000
L	20.000
e	1.000
w	1.000
RO	10.000

Other Parameters:

Y	20.000
CO	10.000
alpha	0.889
sigma	0.500
UUO	1.417

UO	63.496
fullincome	90.0000
Unormalized	90.0000

Other Parameters:

	R^	e^
U^	-0.111111111	0.222222
Le^	-0.111111111	0.111111
C^	-0.111111111	1.111111
L^	0.444444444	-0.44444
y^	0.444444444	0.555556
W^	0	1

Simulation 2:

Free parameters:

Le	90.000
L	10.000
E	1.000
T	0.200
RO	0.100

Other Parameters:

Y	10
CO	8
Alpha	0.918367
UUO	1.326759
UO	73.86422
Fullincome	98
Unomarlized	98
Wr	0.8
dUnormalized/dC	1

Results:

	e^	R^	dt
U^	0.029478458	-0.11111	0.192744
Le^	0.111111111	-0.11111	0.111111
C^	-0.888888889	-0.11111	1.111111

L^	-1	1	-1
y^	0	1	-1
wr^	1	0	-1
TR^	0	1	0

Free Parameters:

Le	10.000
L	90.000
E	1.000
T	0.200
RO	0.100

Other Parameters:

Y	90
CO	72
Alpha	0.121951
UUO	1.448886
UO	56.59522
Fullincome	82
Unomarlized	82
Wr	0.8
dUnormalized/dC	1

Results:

	e^	R^	dt
U^	8.12195122	-9	9.878049
Le^	9	-9	9
C^	8	-9	10
L^	-1	1	-1
y^	0	1	-1
wr^	1	0	-1
TR^	0	1	0