German Transfer Problem with Equilibrium in GAMS

Econ 567

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Introduction

Our model is based on the previous research German Transfer Problem. But this time, we have new objectives and findings. First, we are trying to verify the effects of reparations from our previous model. As usual, given a transfer from German to France would lower German utility and increase France utility. Second, we extend analysis on effects of different initial reparations. This time, we set same values of leisure time and labor time as before with different initial values of reparations and test how utility changes. Third, we attempt to find equilibrium after reparation payments based on wage-distortion. Fourth, we use GAMS to optimize consumption and leisure in order to maximize utility. Hence, the core idea of transfer problem is: when a country transfers money to another country without any counterpart, what will happen to the donor's income and utility? Finally, the German government would finance the reparation by using lump sum tax and ad valorem tax discussed in the two simulations.

Model and Simulations

1. Assumptions

1) Two-commodity, one tradable (e.g. toys), the other non-tradable (leisure).
2) Full-employment. All disposable income goes to domestic consumption.
3) Labor is the only factor of output, while individuals could choose to work (labor time) or non-work (leisure time).
4) Utility functions are Cobb-Douglas where leisure share in fullincome is different in two simulations due to wage distortion.
5) Two simulations; each has a unique way to finance reparation.
   Simulation 1: finances reparations with lump sum tax:
   Reparations = Output - Consumption
   Simulation 2: finances reparations with ad valorem tax:
   Reparations = Tax Revenue = Output * Tax ratio

2. Variables and Parameters

- 14 variables:

  Le – leisure time

  La – labor time

  W – wage rate

  C – consumption

  Unormalized – normalized utility

  Y – production of goods

  U – utility

  Zle – proportional change in leisure time

  Zla – proportional change in labor time

  Zw – proportional change in wage

  Zc – proportional change in consumption

  Zunormalized - proportional change in normalized utility

  Zy – proportional change in production of goods

  Zu – proportional change in utility

- 13 Parameters:

Initial values	Assignment
Ro	= 0
Leo	= 0 < leo < 100
Lao	= 100-leo

Eo	= 1
wo	= eo
Yo	= eo*lao
Co	= Yo-ro
Leshareo	= Leo / (leo+co)
Cshareo	= 1-leshareo
Fullincomeo	= Leo+co
Uo	= leo**leshareo*co**cshareo
Uuo	= fullincomeo/uo
Unormalizedo	= uuo*uo

3. Basic Equations:

No.	LHS	=	RHS
E1	unormalized	=	uuo*le**leshareo*c**cshareo
E2	le+la	=	100
E3	y	=	eo*la
E4	ro	=	y-c

E5	w	=	eo
E6	u	=	unormalized/uuo
E7	w	=	(c*lesharео)/(le*csharео)
E8	zle	=	100*le/leo-100
E9	zla	=	100*la/lao-100
E10	zw	=	100*w/wo-100
E11	zc	=	100*c/co-100
E12	zunormalized	=	100*unormalized/unormalizedo-100
E13	zy	=	100*y/yo-100
E14	zu	=	100*u/uo-100

4.Simulation 1

CASE 1: leo = 80, lao = 20

	ro = 0	**ro = 0.01**	**ro = 0.5**
le	80.000	79.992	79.600
la	20.000	20.008	20.400
w	1.000	1.000	1.000

c	20.000	19.998	19.900
unormalized	100.000	99.990	99.500
y	20.000	20.008	20.400
u	**60.629**	**60.623**	**60.326**
zle	.	-0.010	-0.500
zla	.	0.040	2.000
zw	.	.	.
zc	.	-0.010	-0.500
zunormalized	.	-0.010	-0.500
zy	.	0.040	2.000
zu	.	-0.010	-0.500

- In case 1, we set initial leisure time = 80, labor time = 20.
- Increase exogenous variable ro from 0 to 0.5, leisure time decreases from 80 to 79.6, while labor time increases from 20 to 20.4
- The only determinant of wage rate is working efficiency. Since we fix working efficiency e=1, wage remains constant.
- Consumption decreases from 20 to 19.9, while production output increases from 20 to 20.4.
- Consumer utility is adversely affected by increases in reparation.
- A notable observation is that the ratio of leisure (le) to consumption (c) remains constant to be 4. This result suggests that an increase of reparation leads to an inward and parallel shift of budget constraint of leisure and consumption. Therefore, this result shows that German consumers' taste is Homothetic, so that at different budget levels, MRS of consumer indifference curve remains constant at equilibrium where utility is maximized.

<u>CASE 2:</u> leo = 50, lao = 50

	ro = 0	**ro = 0.01**	**ro = 0.5**
le	50.000	49.995	49.750
la	50.000	50.005	50.250
w	1.000	1.000	1.000
c	50.000	49.995	49.750
unormalized	100.000	99.990	99.500
y	50.000	50.005	50.250
u	**50.000**	**49.995**	**49.750**
zle	.	-0.010	-0.500
zla	.	0.010	0.500
zw	.	.	.
zc	.	-0.010	-0.500
zunormalized	.	-0.010	-0.500
zy	.	0.010	0.500
zu	.	-0.010	-0.500

- In case 2, we set initial leisure time = 50, labor time = 50.
- Increase exogenous variable ro from 0 to 0.5, leisure time decreases from 50 to 49.750, while labor time increases from 50 to 50.25.
- Wage remains constant.
- Since output is determined by labor time and constant efficiency, it increases from 50 to 50.25.

- Consumer utility is adversely affected by increases in reparation.
- A similar notable observation shows that the ratio of leisure (le) to consumption (c) remains constant to be 1. This result suggests that an increase of reparation leads to an inward and parallel shift of budget constraint of leisure and consumption. Therefore, this result shows that German consumers' taste is Homothetic.

CASE 3: leo = 20, lao = 80

	ro = 0	**ro = 0.01**	**ro = 0.5**
le	20.000	19.998	19.900
la	80.000	80.002	80.100
w	1.000	1.000	1.000
c	80.000	79.992	79.600
unormalized	100.000	99.990	99.500
y	80.000	80.002	80.100
u	**60.629**	**60.623**	**60.326**
zle	.	-0.010	-0.500
zla	.	0.002	0.125
zw	.	.	.
zc	.	-0.010	-0.500
zunormalized	.	-0.010	-0.500
zy	.	0.002	0.125

zu		.	-0.010	-0.500

- In case 3, we set initial leisure time = 20, labor time = 80.
- Increase exogenous variable ro from 0 to 0.5, leisure time decreases from 20 to 19.9, while labor time increases from 80 to 80.1.
- Wage remains constant.
- Output increases from 80 to 80.1.
- Consumer utility is adversely affected by increases in reparation.
- A similar notable observation shows that the ratio of leisure (le) to consumption (c) remains constant to be 0.25. This result suggests that an increase of reparation leads to an inward and parallel shift of budget constraint of leisure and consumption. Therefore, this result shows that German consumers' taste is Homothetic.

5.Simulation 2

CASE 1: leo = 80, lao = 20

	ro = 0	**ro = 0.01**	**ro = 0.5**	**ro = 10**
le	80.000	80.000	80.000	80.000
la	20.000	20.000	20.000	20.000
wr	1.000	1.000	0.975	0.5
c	20.000	19.990	19.500	10.000
unormalized	100.000	99.990	99.495	87.055
t	.	5.0000E-4	0.025	0.500

y	20.000	20.000	20.000	20.000
u	**60.629**	**60.623**	**60.322**	**52.780**
zle	.	.	.	.
zla	.	-1.42E-14	-1.42E-14	-1.42E-14
zwr	.	-0.050	-2.500	-50.000
zc	.	-0.050	-2.500	-50.000
zunormalized	.	-0.010	-0.505	-12.945
zy	.	-1.42E-14	-1.42E-14	-1.42E-14
zu	.	-0.010	-0.505	-12.945

Explanation: As indicated in the above table, given reparation from Germany to France would only decrease consumption and wage received because the more reparation is, the more ad valorem tax is being collected, and hence less income would hold by Germans. However, through maximizing utility offer us a result with unchanged leisure and labor time. This situation is attributable to two reasons: For one reason, reparation would only shrink the budget constraint with consumption side in order to maximize utility. For the second reason, leisure is a quasi-linear good for Germen, which means reparation would decrease consumption instead of leisure.

CASE 2: leo = 50, lao = 50

	ro = 0	**ro = 0.01**	**ro = 0.5**	**ro = 10**
le	50.000	50.000	50.000	50.000

la	50.000	50.000	50.000	50.000
wr	1.000	1.000	0.990	0.800
c	50.000	49.990	49.500	40.000
unormalized	100.000	99.990	99.499	89.443
t	.	2.0000E-4	0.010	0.200
y	50.000	50.000	50.000	50.000
u	**50.000**	**49.995**	**49.749**	**44.721**
zle	.	.	.	.
zla	.	.	.	.
zwr	.	-0.020	-1.000	-20.000
zc	.	-0.020	-1.000	-20.000
zunormalized	-1.42E-14	-0.010	-0.501	-10.557
zy	.	.	.	.
zu	-1.42E-14	-0.010	-0.501	-10.557

Explanation: This is a special case where Germen spend even time in labor and leisure. As we know, utility is a convex function of both leisure and labor. Hence, in this case, utility level is very close to a minimum point. More interestingly, normalized utility has the same decreasing rate with utility as reparation increases and also wage received has the same decreasing rate with consumption.

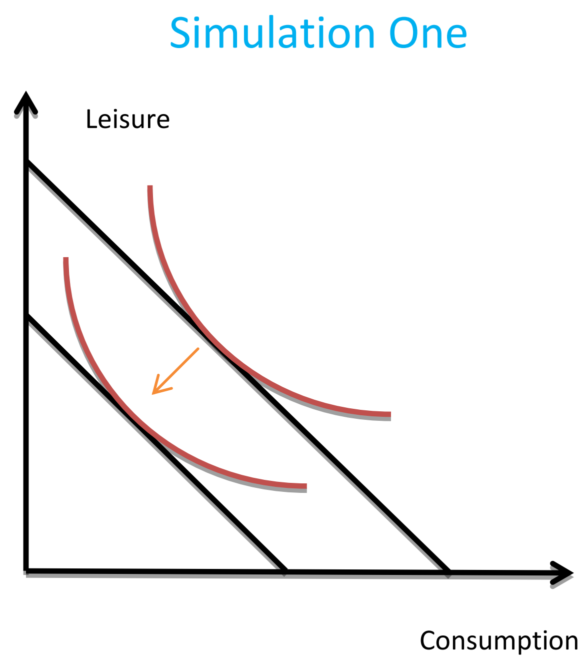
CASE 3: leo = 20, lao = 80

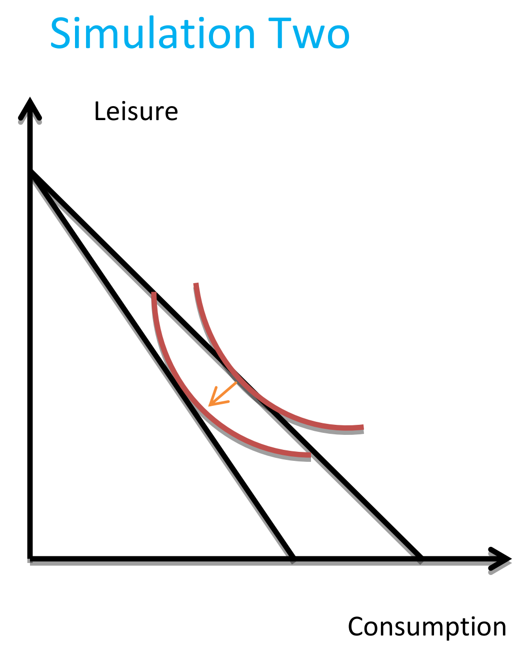
	ro = 0	**ro = 0.01**	**ro = 0.5**	**ro = 10**
le	20.000	20.000	20.000	20.000
la	80.000	80.000	80.000	80.000
wr	1.000	1.000	0.994	0.875
c	80.000	79.990	79.500	70.000
unormalized	100.000	99.990	99.500	89.868
t	.	1.2500E-4	0.006	0.125
y	80.000	80.000	80.000	80.000
u	**60.629**	**60.623**	**60.325**	**54.486**
zle	.	.	.	.
zla	.	.	.	.
zwr	.	-0.013	-0.625	-12.500
zc	.	-0.013	-0.625	-12.500
zunormalized	-1.421E-14	-0.010	-0.500	-10.132
zy	.	.	.	.
zu	-1.421E-14	-0.010	-0.500	-10.132

Explanation: As can be seen from the above table, when we reverse the initial endowment in case one, we got nearly the same result of utility. These small discrepancies maybe resulted from the effects of reparations on consumptions. For example, given a reparation of 10, the utility is slightly larger than that in case 1 because part of reparation is absorbed by ad valorem tax.

6.Conclusion

In both two simulations, we verified the effects of reparations has the same result using excel last time. However, leisure and labor in each case keep unchanged due to the maximized utility result. More strikingly, in simulation 1, we found the ratio of leisure and consumption is a constant because wage does not change. Hence, the slope of the budget constraint does not change and shift the budget constraint inward parallel. However, in simulation 2, the ratio of leisure and consumption is getting larger and larger due to a decreasing wage rate. Therefore, the decline in wage received would result in a higher MRS, the analysis in two simulations can be more easily seen in the following graph.





7.Appendix

<u>Simulation 1 GAMS Code:</u>

```
parameter
leo,lao,eo,wo,ro,co,yo,leshareo,cshareo,sigma,uo,fullincomeo,uuo,unormalizedo;
leo=80;
lao=100-leo;
eo=1;
wo=eo;
ro=0;
yo=eo*lao;
co=yo-ro;
leshareo=leo/(leo+co);
cshareo=1-leshareo;
*alpha-share of leisure in normalized utility
sigma=co/yo;
*sigma_share of consumption in output
fullincomeo=leo+co;
uo=leo**leshareo*co**cshareo;
uuo=fullincomeo/uo;
```

```
unormalizedo=uuo*uo;

variables le,la,w,c,unormalized,y,u,zle,zla,zw,zc,zunormalized,zy,zu;
le.L=leo;la.L=lao;w.L=wo;c.L=co;unormalized.L=unormalizedo;y.L=yo;
u.L=uo;
equations E1,E2,E3,E4,E5,E6,E7,E8,E9,E10,E11,E12,E13,E14;
E1..unormalized=E=uuo*le**leshareo*c**cshareo;
E2..le+la=E=100;
E3..y=E=eo*la;
E4..ro=E=y-c;
E5..w=E=eo;
E6..u=E=unormalized/uuo;
E7..w=E=(c*leshareo)/(le*cshareo);
E8..zle=E=100*le/leo-100;
E9..zla=E=100*la/lao-100;
E10..zw=E=100*w/wo-100;
E11..zc=E=100*c/co-100;
E12..zunormalized=E=100*unormalized/unormalizedo-100;
E13..zy=E=100*y/yo-100;
E14..zu=E=100*u/uo-100;
model leisurereparations/all/;
option limcol=0;
solve leisurereparations using nlp maximizing u;
ro=.01;
solve leisurereparations using nlp maximizing u;
ro=.1;
solve leisurereparations using nlp maximizing u;
```

Simulation 2 GAMS Code:

```
parameter
leo,lao,eo,wro,ro,co,yo,to,leshareo,cshareo,uo,fullincomeo,uuo,unormalizedo;
leo=20;
lao=100-leo;
eo=1;
ro=0;
yo=eo*lao;
to=ro/yo;
co=yo*(1-to);
wro=eo*(1-to);
leshareo=leo*wro/(leo*wro+co);
cshareo=1-leshareo;
```

```
fullincomeo=leo*wro+co;
uo=leo**leshareo*co**cshareo;
uuo=fullincomeo/uo;
unormalizedo=uuo*uo;
variables le,la,wr,c,unormalized,t,y,u,zle,zla,zwr,zc,zunormalized,zy,zu;
*u.L=uo;
le.L=leo;la.L=lao;wr.L=wro;c.L=co;unormalized.L=unormalizedo;y.L=yo;
equations E1,E2,E3,E4,E5,E6,E7,E8,E9,E10,E11,E12,E13,E14,E15;
E1..unormalized=E=uuo*le**leshareo*c**cshareo;
E2..u=E=le**leshareo*c**cshareo;
E3..y=E=eo*la;
E4..ro=E=y-c;
E5..t=E=ro/y;
E6..le+la=E=100;
E7..wr=E=(c*leshareo)/(le*cshareo);
E8..wr=E=eo*(1-t);
E9..zle=E=100*le/leo-100;
E10..zla=E=100*la/lao-100;
E11..zwr=E=100*wr/wro-100;
E12..zc=E=100*c/co-100;
E13..zunormalized=E=100*unormalized/unormalizedo-100;
E14..zy=E=100*y/yo-100;
E15..zu=E=100*u/uo-100;
model leisurereparations/all/;
option limcol=0;
solve leisurereparations using nlp maximizing u;
ro=0.01;
solve leisurereparations using nlp maximizing u;
ro=0.5
solve leisurereparations using nlp maximizing u;
ro=10;
solve leisurereparations using nlp maximizing u;
```