# Factor Taxes in a Closed Economy H-O Model with Flexible Labor 

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## Model

- H-O Model with A capital-intensive and B labor-intensive
- Labor supply depends on after-tax wages, capital supply is exogenous
- $A$ and $B$ are perfect substitutes and consumers have preferences over consumption and leisure
- The government collects revenues from taxes on labor or capital income and uses it to purchase good A
- Consumers gain no utility from government purchases
- The price of good $A$ is a numéraire. Because goods $A$ and $B$ are perfect substitutes, their prices are equal.


## Results - Labor Tax

|  | $\mathbf{G}=\mathbf{0}$ | $\mathbf{G}=\mathbf{2}$ | $\mathbf{G}=\mathbf{4}$ | $\mathbf{G = 6}$ | $\mathbf{G}=\mathbf{8}$ | $\mathbf{G}=\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}$ | 18 | 18.287 | 18.667 | 19.203 | 20.067 | 22.490 |
| $\mathbf{A C}$ | 18 | 16.287 | 14.667 | 13.203 | 12.067 | 12.490 |
| $\mathbf{B}$ | 23.5 | 23.006 | 22.247 | 21.174 | 19.446 | 14.601 |
| $\mathbf{w r}$ | 1 | 0.907 | 0.81 | 0.708 | 0.593 | 0.420 |
| $\mathbf{r r}$ | 1 | 1 | 1 | 1 | 1 | 1 |
| KA | 12 | 12.191 | 12.444 | 12.802 | 13.378 | 14.993 |
| KB | 7.86 | 7.669 | 7.416 | 7.058 | 6.482 | 4.867 |
| $\mathbf{L A}$ | 6 | 6.096 | 6.222 | 6.401 | 6.689 | 7.497 |
| $\mathbf{L B}$ | 15.72 | 15.337 | 14.831 | 14.116 | 12.964 | 9.734 |
| $\mathbf{L E}$ | 2.28 | 2.567 | 2.947 | 3.483 | 4.347 | 6.770 |
| $\mathbf{t L}$ | 0 | 0.093 | 0.19 | 0.292 | 0.407 | 0.580 |
| $\mathbf{U}$ | 13.318 | 12.927 | 12.52 | 12.088 | 11.609 | 10.912 |

## Labor Tax - Notes

- Because consumers gain no utility from government purchases, it is not surprising that utility falls as government spending rises.
- Leisure rises as the tax on labor rises due to the substitution effect - the opportunity cost of leisure is now lower.
- With prices held constant and income falling, wage and rental rates stay constant to maintain budget balance.


## Results - Capital Tax

|  | $\mathbf{G}=\mathbf{0}$ | $\mathbf{G}=\mathbf{2}$ | $\mathbf{G}=\mathbf{4}$ | $\mathbf{G = 6}$ | $\mathbf{G}=\mathbf{8}$ | $\mathbf{G}=\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}$ | 18 | 17.946 | 17.891 | 17.834 | 17.776 | 17.716 |
| $\mathbf{A C}$ | 18 | 15.946 | 13.891 | 11.834 | 9.776 | 7.716 |
| $\mathbf{B}$ | 23.5 | 23.688 | 23.799 | 23.912 | 24.029 | 24.148 |
| $\mathbf{w r}$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $\mathbf{r r}$ | 1 | 0.899 | 0.799 | 0.698 | 0.597 | 0.496 |
| $\mathbf{K A}$ | 12 | 11.964 | 11.927 | 11.889 | 11.851 | 11.811 |
| KB | 7.86 | 7.896 | 7.933 | 7.971 | 8.01 | 8.049 |
| $\mathbf{L A}$ | 6 | 5.982 | 5.964 | 5.945 | 5.925 | 5.905 |
| $\mathbf{L B}$ | 15.72 | 15.792 | 15.866 | 15.941 | 16.019 | 16.099 |
| $\mathbf{L E}$ | 2.28 | 2.226 | 2.171 | 2.114 | 2.056 | 1.996 |
| $\mathbf{t K}$ | 0 | 0.101 | 0.19 | 0.302 | 0.403 | 0.504 |
| $\mathbf{U}$ | 13.318 | 12.93 | 12.536 | 12.135 | 11.726 | 11.31 |

## Capital Tax - Notes

- The picture for the capital tax is very similar to that for the labor tax, except with the sectors reversed.
- The other major difference is that leisure declines instead of rises. Not only is there no substitution effect for labor or capital under a capital tax, but there is an income effect that increases labor.


## Rybcznski's Theorem

- Rybcznski's Theorem holds nicely in these simulations. As the supply (endowment) of labor decreases to compensate for the effect of the labor tax, the production of the capitalintensive good (A) increases and the production of the labor-intensive good (B) decreases.
- The reverse is also true for the capital tax.


## Utility at Various Levels of Spending



## Conclusion

- In this model, the capital tax is superior to the labor tax at all levels of government spending. This holds because the supply of capital is perfectly inelastic and it is better to tax the relatively inelastic factor.
- As the graph on the previous slide shows, the labor tax induces a negative second order effect on utility from the substitution effect. The capital tax induces an imperceptibly small positive second order effect from the income effect.


## Appendix-Parameters

## Parameters

PA Initial price of good A
PB Initial price of good B
KAO Initial capital in good A
LAO Initial labor in good A
KBO Initial capital in good B
LBO Initial capital in good B
KO Capital endowment
LO Initial labor
RO Initial nominal rental rate
WO Initial nominal wage rate
AO Initial amount of good A
BO Initial amount of good B
ALPHA Output elasticity of labor in good A
BETA Output elasticity of labor in good B
GAMMA Utility elasticity of consumption
TAU Utility elasticity of leisure
UO Initial utility
G Target revenue
LEO Initial leisure
AAO Technology factor for A
BBO Technology factor for B;

## Appendix - Parameter Values

```
ALPHA=1/3;
BETA=2/3;
GAMMA=2/3;
TAU=1/3;
PA=1;
PB=1;
WO=1;
RO=1;
AO=18;
BO=(193-641**.5)/(64/9);
KAO=(1-ALPHA)*PA*AO/RO;
LAO=(ALPHA)*PA*AO/WO;
KBO=(1-BETA)*PB*BO/RO;
LBO=(BETA)*PB*BO/WO;
KO=KAO+KBO;
LO=LAO+LBO;
LEO=24-LO;
AAO=AO/(LAO**ALPHA*KAO**(1-ALPHA));
BBO=BO/(LBO**BETA*KBO**(1-BETA));
UO=(AO+BO)**(GAMMA)+(LEO)**TAU;
```


## Appendix - Variables

Variables
A Amount of good A
AC A consumed
AG A consumed by the government
$B$ Amount of good B
R rental rate
rr After-tax rent
W wage
wr After-tax wage
tK Tax rate on capital
tL Tax rate on labor
KA Capital employed to produce good A
KB Capital employed to produce good B
LA Labor employed to produce good A
LB Labor employed to produce good B
LE Leisure
L Total labor supplied
U Utility

## Appendix - Equation Names

```
Equations
Production_A,
Production_B, productivityA,
productivityB,
zeroprofitA,
zeroprofitB,
totalL,
totalK,
taxL,
leisure,
Budj_cons,
Revenue,
govt_consumption,
A_clearing,
prices,
obj;
```


## Appendix - Equations

```
Production_A..A=E=AAO*LA**ALPHA*KA**(1-ALPHA);
Production_B..B=E=BBO*LB**BETA*KB**(1-BETA);
productivityA..W/R=E=ALPHA*KA/((1-ALPHA)*LA);
productivityB..W/R=E=BETA*KB/((1-BETA)*LB);
zeroprofitA..W*LA+R*KA=E=A*PA;
zeroprofitB..W*LB+R*KB=E=B*PB;
totalL..L=E=LA+LB;
totalK..KO=E=KA+KB;
taxL..wr=E=W*(1-tL);
leisure..LE=E=24-L;
Budj_cons..wr*L+R*KO+G=E=PA*A+PB*B;
Revenue..G=E=tL*L;
govt_consumption..AG*PA=E=G;
A_clearing..AG+AC=E=A;
prices..1/wr=E=(GAMMA*(AC+B)**GAMMA*LE)/(TAU*LE**TAU*(AC+B));
obj..U=E=(AC+B)**(GAMMA)+(LE)**(TAU);
```


## Appendix

```
LE.L=LEO;
W.L=WO;
R.L=RO;
A.L=AO;
AC.L=AO;
B.L=BO;
KA.L=KAO;
LA.L=LAO;
KB.L=KBO;
LB.L=LBO;
U.L=UO;
```

L.L=LAO+LBO;
rr.L=RO;
wr.L=WO;
model TAX /ALL/;
G=0;
solve TAX using NLP maximizing U;

