Market Power and Public Policy ECON 465

Cournot, Bertrand, Stackelberg Workout KEY

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1 Short Answer Questions

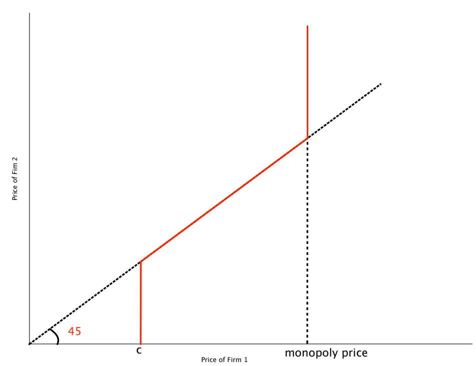
1. The price elasticity of demand for gasoline is -2%. If the government wants to reduce gasoline consumption by 20% how much should it increase the price of gasoline?

   By 10%.

2. The cross-price elasticity of good x as a consequence of an increase in the price of good y is -0.5. Are goods x and y substitutes or complements?

   Cross-price elasticity is negative, hence the goods are complements.

3. Suppose that 2 identical firms produce the same good at marginal cost c and they compete a la Bertrand. Draw the best response of Firm 1.



4. When do we say that a firm has market power? Can you name two sources of market power?

   1. patents

   2. economies of scale

5. Suppose that the elasticity of demand for cars in Germany is -2 and -3 in the U.K., while the marginal cost of these cars is $20,000. How will prices differ in Germany and the U.K.. **Using the formula we derived in class, we know that $p = \frac{MC}{1+1/\epsilon}$. So the price in the UK is 30,000 and in Germany it is 40,000.**

2 Monopoly, Cournot and Stackelberg KEY -

The market demand function for gelato in Summersville is

$$Q^d = 70 - \frac{P}{2}$$

Its cost function for producing gelato is $TC = 5 + 20Q$.

1. What is fixed cost, the variable costs, average costs and marginal costs of producing gelato? Does the cost function of gelato have economies or diseconomies of scale?

   Fixed Costs: FC =**5 (Cost of producing 0 units)** $VC = 20Q$ **(Total Cost - Fixed Costs)** $AC = \frac{TC}{Q} = \frac{5}{Q} + 20$ **Marginal Costs:** $MC = \frac{dTC}{dQ} = 20$

2. Suppose that there is only ONE producer of bathing suits. Find the profit-maximizing quantity and price for bathing suits.

   The output level that maximizes profits for a monopolist satisfies: $MR = MC$.

   First we compute MR**. The inverse demand curve is:**

$$\begin{aligned} Q^d &= 70 - \frac{P}{2} \\ P &= 140 - 2Q \end{aligned}$$

   Revenue R is just total sales and we want to express in terms of Q alone:

$$\begin{aligned} TR &= P(Q) \times Q \\ TR &= (140 - 2Q) \times Q \end{aligned}$$

   Now we can get MR as the derivative of the total revenue function:

$$MR = \frac{dTR}{dQ} = 140 - 4Q$$

   Likewise, MC is just 20. So setting MC=MR we get:

$$\begin{aligned} MC &= MR \\ 20 &= 140 - 4Q \\ 30 &= Q \end{aligned}$$

   The price is just given by the inverse demand function we computed:

$$P(Q) = 140 - 2\,(30) = 80$$

3. Suppose that firm can perfectly price discriminate (first degree price discrimination). How much will is produce? How much will its profits be?

Remember that a perfect price discriminator will sell to all consumers all the way until $P = MC$ (just like perfect competition). Of course price can just be taken from the inverse demand curve $P(Q)$. So

$$\begin{aligned} P(Q) &= MC \\ 140 - 2Q &= 20 \\ 60 &= Q \end{aligned}$$

As for profits, this is a bit more complicated since the firm is charging a different price for each consumer it sells to. Remember that the firm charges the reservation value to each consumer, so the 0's consumer is charged the top intercept of the demand curve, i.e. 140, **while the last customer pays 20. The profits are equal**

Profits= Producer Surplus - Fixed Costs

$$\textbf{Profits=}\ (140 - 20)\left(\frac{60}{2}\right) - 5 = 3595.$$

4. What will be the equilibrium prices and quantities, if there are TWO firms that choose quantities simultaneously? **(Cournot Competition).**

To solve the Cournot Competition Problem we need to get the best-response functions of each firm (reaction functions) and substitute one into the other to get the equilibrium quantities for Firm 1 and Firm 2. Finally, to get the price we replace the total output produced in the market inverse demand:

The inverse demand function is now:

$$P(Q) = 140 - 2Q_1 - 2Q_2$$

Step 1: get TR :Revenue is thus (for firm 1):

$$TR_1 = P(Q_1)Q_1 = (140 - 2Q_1 - 2Q_2)Q_1$$

Step 2: Get MR :To get Marginal Revenue we just take the derivative:

$$MR = \frac{\partial TR_1}{\partial Q_1} = 140 - 4Q_1 - 2Q_2$$

Step 3: Set $MC = MR$:Now we equate marginal costs to marginal revenue:

$$\begin{aligned} 140 - 4Q_1 - 2Q_2 &= 20 \\ Q_1 &= 30 - \frac{Q_2}{2} \end{aligned}$$

We do the same for firm 2 and get (since both firms are the same something quite symmetric):

$$Q_2 = 30 - \frac{Q_1}{2}$$

Finally substitute 2's reaction function into Q_2 in firm 1's reaction function:

$$\begin{aligned} Q_1(Q_2(Q_1)) &= 30 - \frac{Q_2}{2} \\ Q_1(Q_2(Q_1)) &= 30 - \frac{30 - \frac{Q_1}{2}}{2} \\ Q_1 &= 30 - 15 + \frac{Q_1}{4} \\ \frac{3Q_1}{4} &= 15 \\ Q_1 &= 20 \end{aligned}$$

A really good way to check that you've done this right is to see what firm 2 will produce:

$$Q_2(20) = 30 - \frac{20}{2} = 20$$

So both firm's produce the same amount which makes sense since they are exactly the same.

5. Now assume that the first firm gets to choose quantity before the entrant. What are the quantities that these firms will produce and the price in the market **(Stackelberg Competition).** Why are these quantities different?

   For this question we need to solve the game sequentially since firm 1 moves first and firm 2 moves second. As we do in sequential games, let's look at firm 2's best response then we can see how firm 1 would act.

   Firm 2 sets $MR_2 = MC$, which is from the previous question gives:

$$Q_2(Q_1) = 30 - \frac{Q_1}{2}$$

   Now firm 1's revenue will take into account firm 2's reaction. Let's write it out:

$$\begin{aligned} TR_1 &= P(Q_1)Q_1 = (140 - 2Q_1 - 2Q_2(Q_1))Q_1 \\ &= (140 - 2Q_1 - 2\left(30 - \frac{Q_1}{2}\right))Q_1 \\ & (80 - Q_1)Q_1 \end{aligned}$$

So marginal revenue is $MR_1 = 80 - 2Q_1$. Setting MR=MC we obtain:

$$\begin{aligned} MR_1 &= MC \\ 80 - 2Q_1 &= 20 \\ Q_1 &= 30 \end{aligned}$$

So firm 1 produces 246 units. What will firm 2 do in response?

$$Q_2(Q_1) = 30 - \frac{Q_1}{2} = 30 - \frac{30}{2} = 15$$

So firm 2 produces half as much as Firm 1. Why does this happen? By committing to produce a lot, firm 1 will make firm 2 reduce it's own production!